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National Debt and Economic Growth with Externalities and Congestions

Wei-bin Zhang*

Summary

The purpose of this study is to examine the dynamic interdependence between national debt and economic growth with externalities and congestions in public good in a generalized neoclassical three-sector growth model. The economy has two production sectors and one public sector. We describe nonlinear dynamic interactions between growth, economic structural change, capital accumulation, externalities and congestions, and public debt under different combinations of taxes on the industrial good sector, the service sector, the wage income, the rate of interest, consumption of good, and consumption of service. We simulate the motion of the economic system and demonstrate the existence of two equilibrium points. The one equilibrium point with higher national output and capital is stable, and the other one with lower national output and capital is unstable. We examine how the stable and unstable paths react to changes in different parameters over time.

Keywords: growth; economic structure; externalities; congestion; national debt

JEL codes: O41; H23; H63;

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* Associate Dean of International Cooperation and Research Division, Professor of Economics in Ritsumeikan Asia Pacific University (APU), Japan

1. Introduction

Issues related to national debts have become important issues in developing as well as developed economies. The issues are complicated as national debts have complicated relations with GDP, taxes, taxation structures, government's social and economic activities, population structure, and economic growth. Although some theoretical models have been proposed to study relations between national debts and growth, these models mostly deal with very simple economic structures with simplified taxation. The purpose of this study is to address issues related to public debt in neoclassical growth framework.

The main economic growth mechanism is based on neoclassical growth theory. Wealth accumulation is the engine of economic growth. Neoclassical growth theory is developed with the pioneering works of Solow (1956) and Swan (1956). In the standard neoclassical one-sector growth model, capital and labor are substitutes for one another with the result that the long-run growth path of the economy is one of full employment. The model shows that the razor-edge growth path of the Domar model is primarily a result of the particular production function assumption adopted therein and that the need for delicate balancing may not arise when the production function is taken on a different type. The one-sector growth model has been extended and generalized in numerous directions (e.g., Burmeister and Dobell, 1970; Zhang, 2005).

We specially frame our model according to the traditional two-sector economy initially proposed by Uzawa (1961). The Uzawa model generalizes the Solow model by breaking down the one-sector productive system into two sectors using capital and labor. In the Uzawa model, one sector produces capital goods and the other consumption goods. This paper uses an alternative approach to consumer decision to examine structural change within Uzawa's two-sector economy.

This study is concerned with issues related to growth and dynamics of debts. We deal with dynamics of debts by considering government expenditure and different taxes in a competitive economy. We introduce public goods to the growth model and deal with dynamic interdependence between economic growth and public investment. In our approach there are externalities and congestion with regard to public good which affects productivities. There are a few models which treat productive fiscal policy as a determinant of persistent economic growth (Barro, 1990; Turnovsky, 2000, 2004; Gómez, 2008; and Park, 2009). We follow Hochman (1981) and Wijkander (1984) in that the government is concerned with provision of public goods. The government is responsible for the provision of public goods and the government uses different taxes and public debt to finance the public sector. The government is effective in the sense that it minimizes the cost of public goods provision. In our study the government has a set of control measures including the total expenditures and tax rates on the industrial sector's output, the service sector's output, the wage income, the consumption and the interest income. The modeling of this study is also influenced by Lin (2000). It is different from Lin's model in that the Lin's model is developed on the basis of the Uzawa-Lucas two-sector model, while this paper is based on the Uzawa Two-sector model. Lin's model assumes that the government expenditure is spent on education, while in this study the public expenditure is spent on providing public services, which are used by the two production sectors and the household. In Lin's model, there is only one lump sum tax on the household, while in this study the government may tax on the household's wage income, wealth income, consumption, as well as on the two production sectors' outputs. This study takes account of externalities and congestion in public good. Lin's examination focuses on the effects of changes in some parameters on the steady state, while this study focuses on effects of changes in some parameters on the dynamic paths of the economic system as well as on the equilibrium points.

Almost of all recent theoretical literature of dynamic interactions between economic growth and public debts use either the Ramsey framework in continuous time (Cohen and Sachs, 1986; Blanchard and Fischer, 1989; Barro *et al*. 1995; Semmler and Sieveking, 2000; Guo and Harrison, 2004; and Giannitsarou, 2007) or the OLG modeling framework in discrete time (Diamond, 1965; Farmer, 1986; Turnovsky and Sen, 1991; Azariadis, 1993; de la Croix and Michel, 2002; and Chalk, 2000). This model deviates from the mainstreams and follows the model recently developed by Zhang (2016). This study is concerned with effects of externalities and congestion on the economic dynamics, while the model by Zhang is concerned with the traditional neoclassical grow theory in which neither externalities nor congestions is taken into account. The introduction of externalities and congestion causes analytical difficulties. Zhang's 2016 paper integrates the Solow-Uzawa-Diamond neoclassical growth models, while this paper unifies the Solow-Uzawa-Diamond- Eicher and Turnovsky neoclassical growth models with Zhang's approach to household behavior. The rest of the paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the model. Section 4 examines effects of changes in some parameters on the economic system over time. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. The growth model with government debt

The model is built on the basic features of three well-known models, the growth model of Solow (1956), the two-sector growth model of Uzawa (1961), and the growth model with public debt of Diamond (1965). Modelling of household decision applies Zhang's approach (Zhang, 1993, 2005). Our model differs from Zhang's model (Zhang, 2016) mainly in two points. This study introduces congestion and externalities of public goods. Rather than a fixed expenditure of the public sector, this model assumes that the government's expenditure is endogenous. The government expenditure depends on the national output. The economy produces industrial good, consumption good, and public good. It consists of three – industrial, consumer, and public – good sectors. The public sector uses capital and labor as inputs and supplies public services, which are freely available to consumers and producers. The public sector is financially by the government, which taxes the household and the two production sectors. The price of the industrial good is unity. Capital depreciates at a constant exponential rate, δ_k. Technologies of the production sectors are described the Cobb-Douglas production functions. The markets are perfectly competitive and capital and labor are completely mobile between the sectors. All private assets are held by households either as capital or in the form of government debt. We assume that labor is homogeneous and is fixed. We use subscript index, i, s, and p, to denote respectively the industrial, service and public sectors. Let τ_i, τ_s, τ_w, and τ_k, stand for, respectively, the fixed tax rates on the industrial output, the service output, the wage income, and the interest income. We introduce $\bar{\tau}_x \equiv 1 - \tau_x$, where $x = i, s, w, k$. We use $K_j(t)$ and $N_j(t)$ to represent the capital stocks and labor force employed by sector j, $j = i, s, p$, at time t.

The industrial sector

Let $K_j(t)$ and $N_j(t)$ stand for the capital stocks and labor force employed by sector j. We use $F_j(t)$ to represent the output level of sector j. The production function of the industrial sector is given by

$$F_i(t) = \Omega_i(t) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \ \ \alpha_i, \beta_i > 0, \ \ \alpha_i + \beta_i = 1, \quad (1)$$

where α_i, and β_i are parameters and $\Omega_i(t)$ is a function of externalities, public service and congestion. We specify $\Omega_i(t)$ as follows

$$\Omega_i(t) = A_i F_p^{\theta_{pi}} K_i^{\theta_{ei}}(t) \left(\frac{K_p(t)}{K_i(t)} \right)^{\theta_{ci}}, \ \ A_i, \theta_{pi}, \theta_{ei}, \theta_{ci} \ge 0,$$

where $F_p^{\theta_{pi}}(t)$ measures the effect of public service on the industrial sector's productivity, $K_i^{\theta_{ei}}(t)$ the effect of externalities, and $\left(K_p(t)/K_i(t)\right)^{\theta_{ci}}$ the effect of congestion of public goods. Similar to Eicher and Turnovsky (2000), we interpret that when $\theta_{ei} = \theta_{ci} = 0$, there is neither congestion nor externality. The nonrival and nonexcludable public service is available equally to each agent, independent of the usage of others. Obviously, this is a limited case as most of public services are subject to some degree of congestion. We take account of congestion effects by the term $\left(K_p(t)/K_i(t)\right)^{\theta_{ci}}$. The term implies that a rise in the private capital reduces the efficiency of public services for a fixed level of public capital (Gómez, 2008). Markets are competitive. We use $w(t)$ and $r(t)$ to represent, respectively, the wage rate and rate of interest. The marginal conditions for the industrial sector are

$$r_\delta(t) = \alpha_i \, \bar{\tau}_i \, \Omega_i(t) k_i^{-\beta_i}(t), \;\; w(t) = \beta_i \, \bar{\tau}_i \quad (2)$$

where $k_i(t) \equiv K_i(t)/N_i(t)$, and $r_\delta(t) \equiv r(t) + \delta_k$.

Service sector

The service production needs two inputs capital $K_s(t)$ and labor force $N_s(t)$. The production function of the service sector is

$$F_s(t) = \Omega_s(t) K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \;\; \alpha_s, \beta_s > 0, \;\; \alpha_s + \beta_s = 1, \quad (3)$$

where α_s and β_s are parameters and $\Omega_s(t)$ is a function of externalities, public service and congestion. We specify $\Omega_s(t)$ as follows

$$\Omega_s(t) = A_s \, F_p^{\theta_{ps}}(t) K_s^{\theta_{es}}(t) \left(\frac{K_p(t)}{K_s(t)} \right)^{\theta_{cs}}, \;\; A_s, \theta_{ps}, \theta_{es}, \theta_{cs} \geq 0.$$

The prices are determined by market mechanism. The marginal conditions for the service sector are

$$r_\delta(t) = \alpha_s \, \bar{\tau}_s \, \Omega_s(t) p(t) k_s^{\alpha_s - 1}(t), \;\; w(t) = \beta_s \, \bar{\tau}_s \, \Omega_s(t) p(t) k_s^{\alpha_s}(t), \quad (4)$$

where $k_s(t) \equiv K_s(t)/N_s(t)$.

The current and disposable incomes

This study uses an alternative approach for modeling consumers' behavior by Zhang (2005). First, we use $\bar{k}(t)$ and $d(t)$ to represent respectively the real wealth and government debt owned by a representative household. The current income is

$$y(t) = \bar{\tau}_k \, r(t) \bar{k}(t) + \bar{\tau}_w \, w(t) + r(t) d(t), \quad (5)$$

where $r(t)\bar{k}(t)$ is the interest payment and $w(t)$ the wage payment. The total value of the wealth that a consumer can sell to purchase goods and to save is equal to $\bar{k}(t) + d(t)$. The disposable income at any point in time is

$$\hat{y}(t) = y(t) + \bar{k}(t) + d(t). \quad (6)$$

The disposable income is used for saving and consumption. At time t the consumer has the total amount of income equaling $\hat{y}$ to distribute between consuming and saving. In the growth literature, for instance, in the Solow model, the saving is out of the current income, $y(t)$, while in this study the saving is out of the disposable income which is dependent both on the current income and the value of wealth.

The budget and optimal decision

At each point in time, the representative household uses the total available budget to consume services $c_s(t)$, industrial goods $c_i(t)$, and saving $s(t)$. The budget constraint is

$$(1 + \tilde{\tau}_s) p(t) c_s(t) + (1 + \tilde{\tau}_c) c_i(t) + s(t) = \hat{y}(t), \quad (7)$$

where $\tilde{\tau}_s$ and $\tilde{\tau}_c$ are respectively the tax rates on the consumption of services and industrial good. Equation (7) means that the consumption and saving exhaust the consumers' disposable personal income. We assume that utility level $U(t)$ of the household is dependent on $c_s(t)$, $c_i(t)$ and $s(t)$ as follows

$$U(t) = \theta \, F_p^{d_0}(t) c_s^{\gamma_0}(t) c_i^{\xi_0}(t) s^{\lambda_0}(t), \;\; d_0, \gamma_0, \xi_0, \lambda_0 > 0,$$

in which θ and d_0 are parameters, and γ_0, ξ_0, and λ_0 are a typical person's elasticity of utility with regard to services, industrial good, and saving. We call γ_0, ξ_0, and λ_0 propensities to consume services, to consume industrial good, and to hold wealth, respectively. Maximizing $U(t)$ subject to (7) yields

$$c_s(t) = \frac{\gamma \, \hat{y}(t)}{p(t)}, \;\; c_i(t) = \xi \, \hat{y}(t), \;\; s(t) = \lambda \, \hat{y}(t), \quad (8)$$

where

$$\gamma \equiv \frac{\rho \gamma_0}{1 + \tilde{\tau}_s}, \;\; \xi \equiv \frac{\rho \xi_0}{1 + \tilde{\tau}_c}, \;\; \lambda \equiv \rho \lambda_0, \;\; \rho \equiv \frac{1}{\gamma_0 + \xi_0 + \lambda_0}.$$

The change in wealth

The household's total wealth is given by $a(t) = \bar{k}(t) + d(t)$. According to the definition of $s(t)$, the wealth accumulation for the household is

$$\dot{a}(t) = s(t) - a(t). \tag{9}$$

This equation states that the change in wealth equals the saving minus the dissaving.

The public sector

The public sector is financially supported by the government. The capital stocks and workers employed by the public sector are paid at the same rates that the private sector pays the services of these factors. The production of public services is to combine capital $K_p(t)$ and labor force $N_p(t)$ as follows

$$F_p(t) = A_p K_p^{\alpha_{0p}}(t) N_p^{\beta_{0p}}(t), \ \ \alpha_{0p}, \beta_{0p}, A_p > 0. \tag{10}$$

Let $Y_p(t)$ stand for the government's expenditure on supplying public goods and services. We define the national output by

$$Y(t) = F_i(t) + p(t)F_s(t).$$

Different from Zhang (2016) where the government expenditure is constant over time, this study assumes that $Y_p(t)$ is proportional to the national output as follows

$$Y_p(t) = \tau\left[F_i(t) + p(t)F_s(t)\right], \tag{11}$$

where $\tau\ (<1)$ is a non-negative parameter. This implies that the government is endogenously determined. The public sector has the following budget constraint

$$w(t)N_p(t) + r_\delta(t)K_p(t) = Y_p(t). \tag{12}$$

Maximization of public services under the budget constraint yields

$$r_\delta(t)K_p(t) = \alpha_p Y_p(t), \ \ w(t)N_p(t) = \beta_p Y_p(t), \tag{13}$$

in which

$$\alpha_p \equiv \frac{\alpha_{0p}}{\alpha_{0p} + \beta_{0p}}, \ \ \beta_p \equiv \frac{\beta_{0p}}{\alpha_{0p} + \beta_{0p}}.$$

The government finances current spending by collecting taxes and issuing interest-bearing debt. Let $T_p(t)$ stand for the government's tax income. The income comes from taxing the two sectors, the ownership of wealth, and consumption. We have

$$T_p(t) = \tau_i F_i(t) + \tau_s\, p(t)F_s(t) + \tau_k\, r(t)\bar{k}(t)N +$$
$$+ \tilde{\tau}_s\, p(t)c_s(t)N + \tilde{\tau}_i\, c_i(t)N. \tag{14}$$

The dynamics of debt

The government debt follows the following dynamics

$$\dot{D}(t) = r(t)D(t) + Y_p(t) - T_p(t). \tag{15}$$

The household owning the government debt

$$d(t)N = D(t).$$

Demand of and supply for services

The equilibrium condition for services is

$$c_s(t)N = F_s(t). \tag{16}$$

Full employment of capital and labor

The total capital stocks employed by the country $K(t)$ is employed by the three sectors. The full employment of labor and capital is represented by

$$K_i(t) + K_s(t) + K_p(t) = K(t),$$
$$N_i(t) + N_s(t) + N_p(t) = N. \tag{17}$$

We have thus built the dynamic growth model.

3. The Dynamics of the Economy

The appendix shows that the motion of the economic system is determined by two differential equations with $z(t)$ and $\bar{k}(t)$ as variables, where $z(t) \equiv w(t)/(r(t) + \delta_k)$. The following lemma shows how we can simulate the dynamic system with computer.

Lemma

The motion of $\bar{k}(t)$ and $z(t)$ is determine by determined by the following two differential equations

$$\dot{\bar{k}}(t) = \tilde{\Psi}(z(t), \bar{k}(t)),$$
$$\dot{z}(t) = \hat{\Psi}(z(t), \bar{k}(t)), \tag{18}$$

in which $\tilde{\Psi}$ and $\hat{\Psi}$ are functions of $\bar{k}(t)$ and $z(t)$ defined in the appendix. We determine all the other variables as functions of $\bar{k}(t)$ and $z(t)$ as follows: $D(t)$ by (A17) → $r_\delta(t)$ by (A16) → $r(t) = r_\delta(t) - \delta_k$ → $K_s(t)$ by (A12) → $K_i(t)$ by (A13) → $K_s(t)$ by (A14) → $k_j(t) = z(t)/\bar{\alpha}_j$, $j = i, s, p$ → $N_j(t) = K_j(t)/k_j(t)$ → $w(t) = z(t) r_\delta(t)$ → $\Omega_i(t)$ and $\Omega_s(t)$ by the specified forms → $F_j(t)$ by the specified forms → $p(t)$ by (4) → $\hat{y}(t) = p(t) F_s(t)/\gamma N$ → $c_s(t), c_i(t)$, and $s(t)$ by (8) → $T_p(t)$ by (14) → $K(t) = \bar{k}(t) N$ → $Y_p(t)$ by (10) → $a(t) = \bar{k}(t) + d(t)$.

The lemma implies that the motion of economic system at any point in time can be described as functions of the two variables, $\bar{k}(t)$ and $z(t)$. If we determine the motion of $\bar{k}(t)$ and $z(t)$, the procedure gives the motion of the whole system. As the expressions are too tedious, it is difficult to get explicit conclusions. For interpretation, we simulate the model. We specify parameter values as follows

$$N = 400,\ \tau = 0.01,\ A_i = 1.2,\ A_s = 1.1,\ A_p = 0.6,\ \alpha_i = 0.31,\ \alpha_s = 0.36,\ \alpha_{0p} = 0.2,$$
$$\beta_{0p} = 0.4,\ \lambda_0 = 0.9,\ \xi_0 = 0.2,\ \gamma_0 = 0.15,\ \theta_{es} = 0.08,\ \theta_{is} = 0.01,\ \theta_{pi} = 0.07,\ \theta_{ps} = 0.06,$$
$$\theta_{ps} = 0.06,\ \theta_{ei} = 0.04,\ \theta_{cs} = 0.06,\ \theta_{ci} = 0.04,\ \tilde{\tau}_s = 0.01,\ \tilde{\tau}_i = 0.03,\ \tau_k = 0.03,$$
$$\tau_w = 0.01,\ \tau_i = 0.03,\ \tau_s = 0.03,\ \delta_k = 0.05. \tag{19}$$

The population is 400. The government spends 1% of the national output on supplying public goods. The propensity to save is 0.9. The propensity to consume industrial good is 0.2, which is much higher than the propensity to consume services. The tax rates on consumption goods, service and interest income are respectively 8 percent, 3 percent, and 13 percent. The tax rates on the two sectors are 3 percent. The depreciation rate of physical capital is fixed at 5 percent. We also take account of externalities and congestions by assuming θ_{mh} nonnegative. Although the choice of these parameter values is not specified on any country's economy, the specified values enables us to see some behavior of the economic system and illustrate economic mechanisms.

By (18) equilibrium is determined by

$$\tilde{\Psi}(z, \bar{k}) = 0,\ \hat{\Psi}(z, \bar{k}) = 0. \tag{20}$$

Equations (20) have two equilibrium points. One equilibrium point is as follows

$$\theta = -5.54,\ Y = 1839.3,\ D = -10195,\ K = 13187.6,\ Y_p = 18.4,\ N_i = 294.7,\ N_s = 101.3,$$
$$N_p = 4,\ K_i = 9123.8,\ K_s = 3925.3,\ K_p = 138.6,\ F_i = 1342,\ F_s = 701.4,\ F_p = 2.8,$$
$$w = 3.05,\ r = -0.0058,\ p = 0.71,\ a = 7.48,\ \bar{k} = 32.97,\ c_i = 1.63,\ c_s = 1.75, \tag{21}$$

where $\theta \equiv D/Y$ is the ratio of debt to the national output. The eigenvalues at this equilibrium is

$$\{-0.26,\ -0.03\}.$$

The equilibrium point is stable. We illustrate the motion of system near this equilibrium with the following initial condition as in Figure 1. From the figure we see that the system approaches to its equilibrium point in the long term.

$$\bar{k}(0) = 32,\ \ z(0) = 68.$$

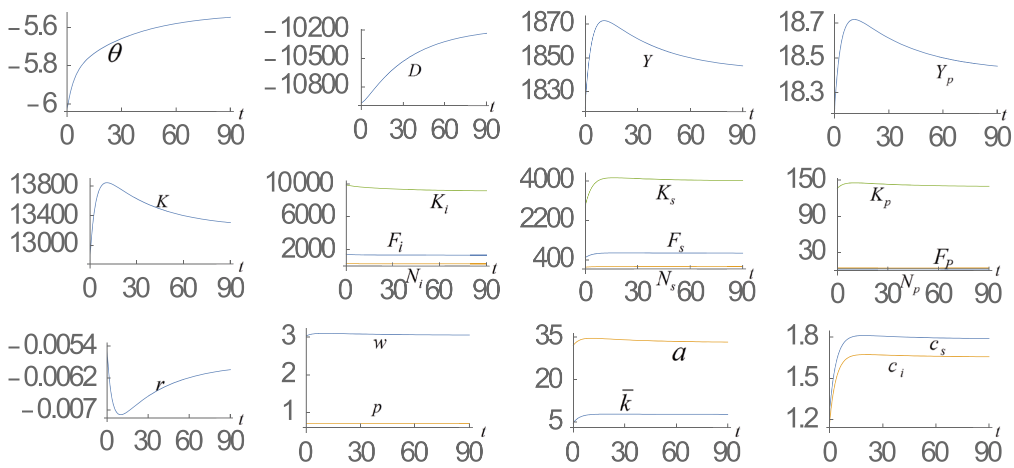

Figure 1. *The Motion of the Economic System near the Stable Equilibrium Point*

We refer the equilibrium as the high equilibrium point and the motion of the system near this point as the stable path of development. The government debt is negative along the stable path. The rate of interest is negative. The cost of capital for firms $r(t) + \delta_k$ is positive. The other equilibrium point is as follows

$$\theta = 0.39,\ Y = 866.8,\ D = 336.7,\ K = 1670.0,\ Y_p = 8.67,\ N_i = 250.9,\ N_s = 145.1,$$
$$N_p = 4.01,\ K_i = 959.2,\ K_s = 694.4,\ K_p = 17.3,\ F_i = 534,\ F_s = 393.5,\ F_p = 1.86,$$
$$w = 1.42,\ r = 0.18,\ p = 0.85,\ a = 5.02,\ \bar{k} = 4.18,\ c_i = 1.09,\ c_s = 0.98. \quad (22)$$

It is straightforward to calculate the two eigenvalues of the system as follows

$$\{-0.19,\ 0.14\}.$$

The equilibrium point is unstable. We plot the motion of the economic system with the following initial condition

$$\bar{k}(0) = 4,\ \ z(0) = 8.$$

As the system does not approach its equilibrium point in the long term, we plot the motion only with a short period of time.

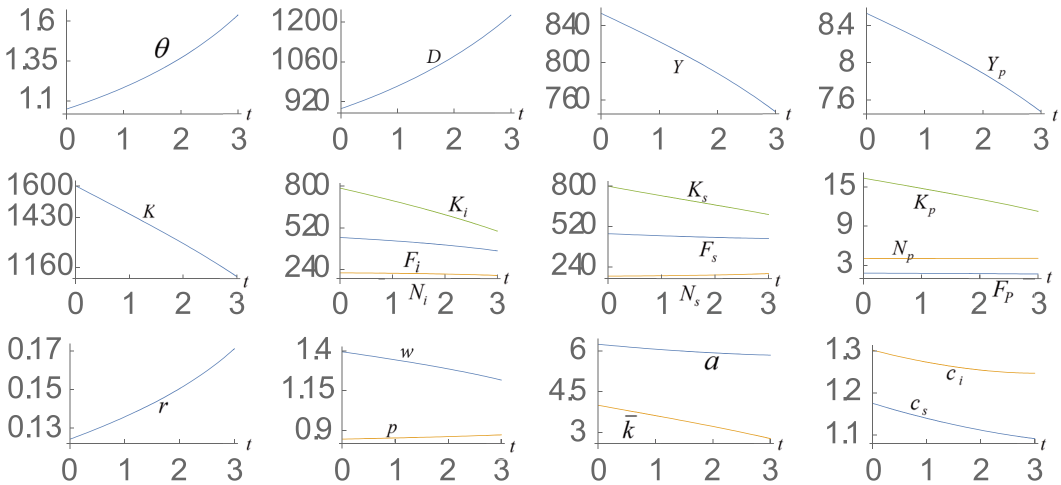

Figure 2. *The Motion of the Economic System near the Unstable Equilibrium Point*

We refer this equilibrium as the low equilibrium point and the motion of the system near this point as the unstable path of development. The system has two equilibrium points. The stable one has higher income and output levels, higher wage and lower price of services than the unstable one. Although the two equilibrium points have the same tax rates and expenditure rate, the lower equilibrium has debt and the higher equilibrium has credit. At the higher equilibrium point, the government has more tax incomes as the output levels, consumption levels and wealth level are higher. It is straightforward to show that if we neglect taxation and debt in the model, the system has a stable unique equilibrium (Zhang, 2005). The introduction of dynamics of the public debt with externalities and congestion leads to multiple equilibria in the neoclassical growth model. It should be noted that that some neoclassical growth models with debts have saddle points (e.g., Turnovsky and Sen, 1991, Lin, 2000). For this kind of models, it is important to follow shifts of paths with changes in public policies and other parameters since the focus on unstable steady states does not provide enough information for behavior of the system. Different policies for stabilizing government debts are suggested in the literature. For instance, according to Michel et al. (2010: 925), “To operationalize the notion of unstable government debt dynamics, we consider steady states which are locally unstable under the assumption of a permanently balanced primary budget. However, the economy can be stabilized at these steady states if one allows for appropriate budgetary adjustments. For tractability, we consider debt stabilizing rules that specify these adjustments as a linear function of the two state variables of the model (physical capital and real government debt). Moreover, we assume that such adjustments can be brought about by two different instruments (government consumption or a lump-sum tax on young agents).” In traditional growth models with debt equilibria can be locally unique or indeterminate, depending on whether the government uses income taxes, consumption taxes or other policies (e.g., Judd, 1987; Turnovsky,1990; Schmitt-Grohe and Uribe, 1997; and Mankiw and Weinzierl, 2006).

4. Comparative Dynamic Analysis

The previous section plots the motion of the variables. As we have shown how to simulate the motion of the system, it is straightforward to make comparative dynamic analysis. We introduce a variable $\overline{\Delta} x_j(t)$ which stands for the change rate of the variable, $x_j(t)$, in percentage due to a change in the parameter value.

4.1. A rise in the government's expenditure rate

We first increase the government's expenditure rate as follows: $\tau: 0.01 \Rightarrow 0.012$. As the system has two equilibrium points, we contact the comparative dynamic analysis separately for the two paths.

Impact on the stable path

We illustrate the effects in Figure 3. As the expenditure rate is increased, the debt is reduced and output is increased. The ratio of the debt to the national output is reduced over time. The government expenditure is increased. The output levels of the three sectors are increased. The tax income is increased. The wage rate and rate of interest are enhanced and the price of service is reduced. The consumption levels and wealth levels are increased. Some of the labor force is shifted from the industrial sectors to the other two sectors.

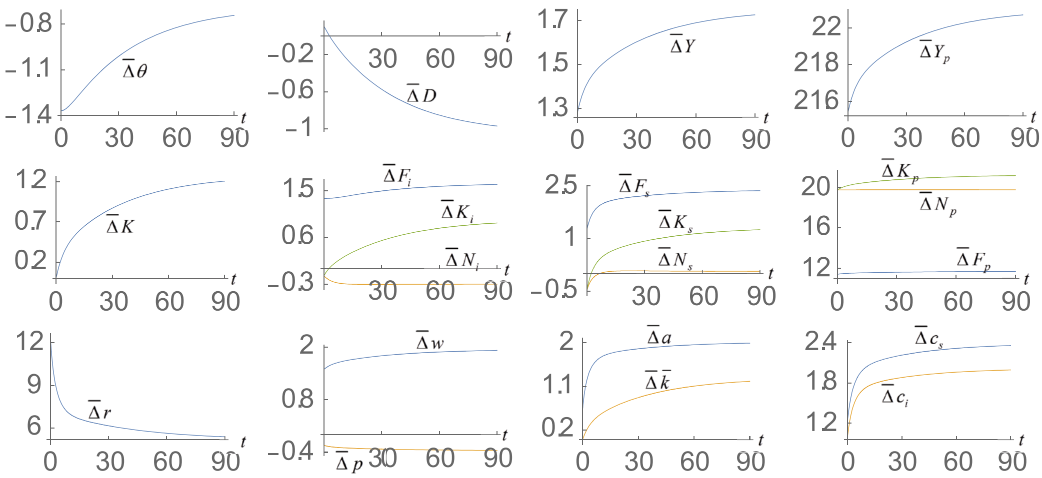

Figure 3. *A Rise of the Expenditure Rate for the Stable Path*

Impact on the unstable path

We plot the effects in Figure 4. As the unstable path will not approach the equilibrium point except the initial condition is located on some special path. The difficulty of describing possible dynamic adjustments is that one has to be follow dynamics under different exogenous changes. As our model can simulate the motion of the dynamic system, we can examine the dynamics of the whole system under any combination of fiscal policies and other parameters. As Michel et al. (2010: 923) emphasize, "Unstable government debt dynamics can typically be stabilized around a certain target level of debt by appropriate budgetary adjustments. To achieve the needed budgetary corrections a government can normally adjust a broad range of fiscal instruments, like government spending, taxes or transfers. Yet, depending on the timing of actions and the particular instrument (or subset of instruments) that gets adjusted, successful stabilizations can be associated with a broad range of possible sequences of the budgetary balance during the adjustment period." Due to the properties of unstable equilibrium it is difficult to predict the directions of adjustment. Figure 4 shows how the system adjusts to the change in the expenditure. In the short term the debt and debt ratio are increased.

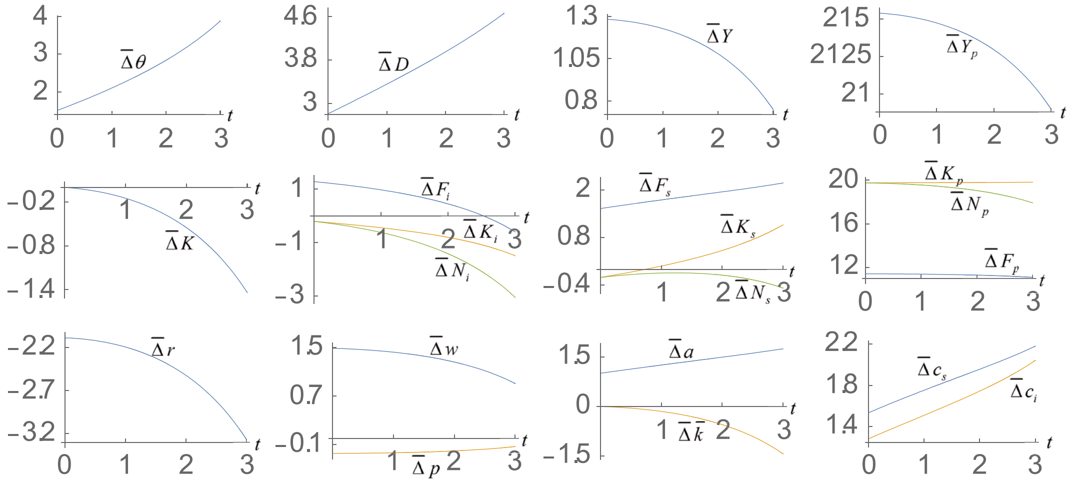

Figure 4. *A Rise of the Expenditure Rate for the Unstable Path*

The impact on the unstable equilibrium point is provided as (23). We did not list the effects on the stable path as they can be read from Figure 3. In the new equilibrium point the debt ratio is reduced. The national output and debt are increased. The public sector enlarges its production scale, employing more labor and capital. As the public sector supplies more public goods, the productivities of the two production sectors are increased. Accordingly, the two sectors' output levels are enhanced. The physical wealth is increased, which results in the increases of the capital stocks employed by all the sectors. The increased wealth is associated with the falling rate of interest and increasing wage rate. In association with the falling price of service, the consumption level of service is increased. It should be noted that there are many studies concerning conditions for debt neutrality (Yaari, 1965; Blanchard, 1984, 1985; and Buiter, 1988). The model by Barro (1974) demonstrates the Ricardian equivalence hypothesis which implies that any mix of public debt and a lump-sum tax to finance government lump sum transfers has no real effect. In our approach as the government expenditure affects economic productivities a change in the government policy will affect the equilibrium of the economic system.

$$\bar{\Delta}\theta = -1.36,\ \bar{\Delta}Y = 3.22,\ \bar{\Delta}D = 1.82,\ \bar{\Delta}K = 5.36,\ \bar{\Delta}Y_p = 23.9,\ \bar{\Delta}N_i = -0.8,\ \bar{\Delta}N_s = 19.8,$$
$$\bar{\Delta}N_p = 4.4,\ \bar{\Delta}K_i = 6.2,\ \bar{\Delta}K_s = 26.1,\ \bar{\Delta}K_p = 26.1,\ \bar{\Delta}F_i = 2.57,\ \bar{\Delta}F_s = 5,\ \bar{\Delta}F_p = 12.6,$$
$$\bar{\Delta}w = 3.4,\ \bar{\Delta}r = -2.5,\ \bar{\Delta}p = -0.72,\ \bar{\Delta}a = 4.8,\ \bar{\Delta}\bar{k} = 5.4,\ \bar{\Delta}c_i = 4.3,\ \bar{\Delta}c_s = 5. \tag{23}$$

4.2. A rise in the tax rate on the industrial sector

We now increase the tax rate on the industrial as follows: $\tau_i : 0.03 \Rightarrow 0.04$.

Impact on the stable path

The impact is plotted in Figure 5. The national debt falls. The national output falls initially and rises in the long term. The output levels of the three sectors are increased. The rate, wage rate, and rate of interest are all reduced. The consumption level of industrial good falls and that of service rises.

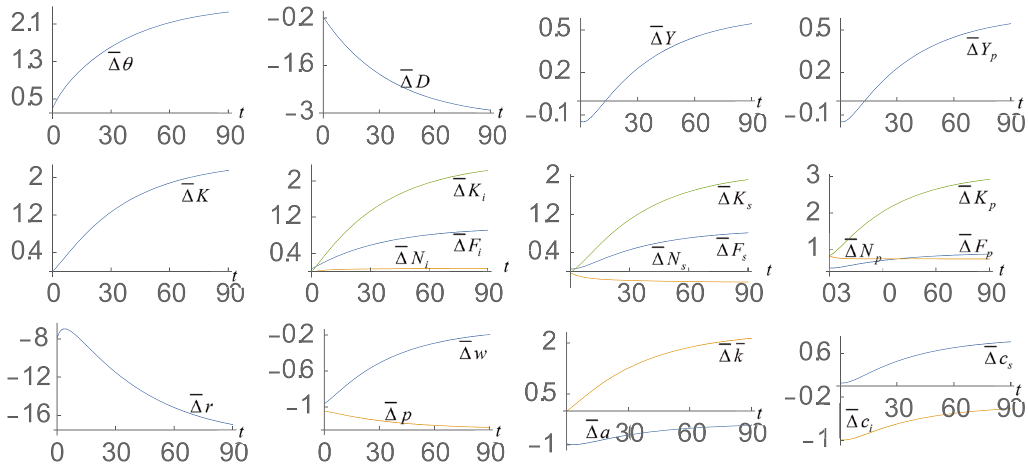

Figure 5. *A Rise of the Tax Rate on the Industrial Sector for the Stable Path*

Impact on the unstable path

The short-run impact is plotted in Figure 6. The ratio and debt fall. The national output falls initially and rises soon. The output levels of the industrial sector and public sector are increased.

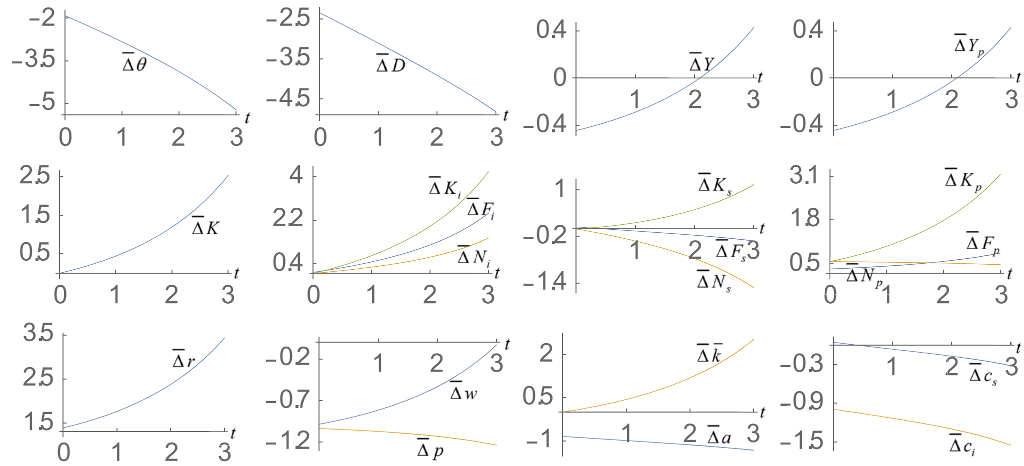

Figure 6. *A Rise of the Tax Rate on the Industrial Sector for the Unstable Path*

We list the effects on the system in (24) as follows

$$\overline{\Delta}\theta = 13.9,\ \overline{\Delta}Y = -0.93,\ \overline{\Delta}D = 12.8,\ \overline{\Delta}K = -1.56,\ \overline{\Delta}Y_p = -0.93,\ \overline{\Delta}N_i = -.17,$$
$$\overline{\Delta}N_s = 2,\ \overline{\Delta}N_p = 0.68,\ \overline{\Delta}K_i = -2.88,\ \overline{\Delta}K_s = 0.25,\ \overline{\Delta}K_p = -1.06,\ \overline{\Delta}F_i = -1.74,$$
$$\overline{\Delta}F_s = 1.31,\ \overline{\Delta}F_p = 0.06,\ \overline{\Delta}w = -1.6,\ \overline{\Delta}r = 0.18,\ \overline{\Delta}p = -0.93,\ \overline{\Delta}a = 0.85,$$
$$\overline{\Delta}\bar{k} = -1.56,\ \overline{\Delta}c_i = 0.38,\ \overline{\Delta}c_s = 1.32. \tag{24}$$

4.3. A rise in the propensity to save

We now increase the propensity to save as follows: $\lambda_0 : 0.9 \Rightarrow 0.91$.

Impact on the stable path

The impact of the rise of the propensity to save is plotted in Figure 7. The national debt and national output are increased, while the ratio is reduced. The output levels of the three sectors are enhanced in the long term. The consumption levels are slightly increased in the long term. The rate of interest and price of service fall and the wage rate rises.

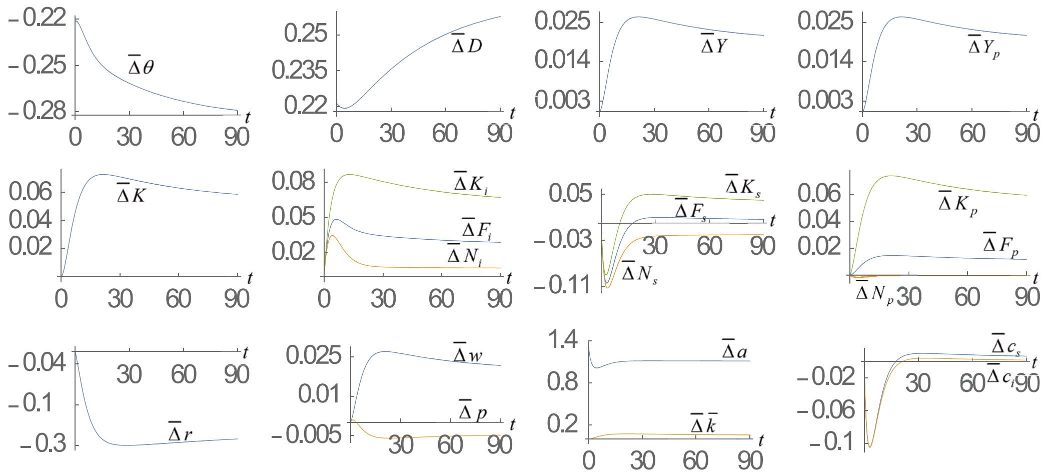

Figure 7. *A Rise of the Propensity to Save for the Stable Path*

Impact on the unstable path

The impact on the system for the unstable path is plotted in Figure 8. The national debt rises and national output falls. The ratio is enhanced in short term.

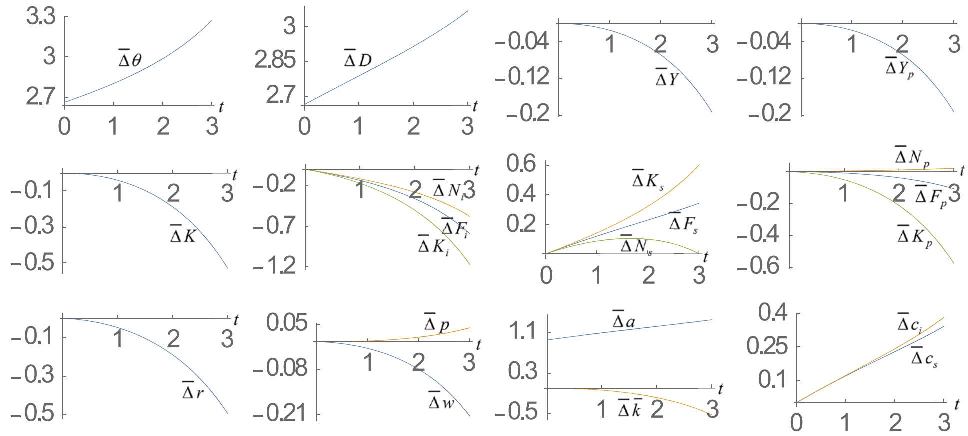

Figure 8. *Figure 7. A Rise of the Propensity to Save for the Unstable Path*

The impact on the unstable equilibrium point is listed in (25). The national debt and national output are increased.

$$\overline{\Delta}\theta = 3.8,\ \overline{\Delta}Y = 1.34,\ \overline{\Delta}D = 5.2,\ \overline{\Delta}K = 3.7,\ \overline{\Delta}Y_p = 1.3,\ \overline{\Delta}N_i = -0.6,$$
$$\overline{\Delta}N_s = 1,\ \overline{\Delta}N_p = 0.03,\ \overline{\Delta}K_i = 3,\ \overline{\Delta}K_s = 4.7,\ \overline{\Delta}K_p = 3.7,\ \overline{\Delta}F_i = 0.7,$$
$$\overline{\Delta}F_s = 2.7,\ \overline{\Delta}F_p = 0.7,\ \overline{\Delta}w = 1.3,\ \overline{\Delta}r = -3.2,\ \overline{\Delta}p = -0.33,\ \overline{\Delta}a = 3.96,$$
$$\overline{\Delta}\bar{k} = 3.7,\ \overline{\Delta}c_i = 2.3,\ \overline{\Delta}c_s = 2.7. \tag{25}$$

4.4. A rise in the total factor productivity of the industrial sector

We now increase the total factor productivity of the industrial sector as follows: $A_i : 1.2 \Rightarrow 1.25$.

Impact on the stable path

The national output is increased and the debt is reduced. The ratio is reduced. The total expenditure is increased. The output levels of the three sectors and total capital are enhanced. The wage rate and the consumption levels of industrial good and good are increased.

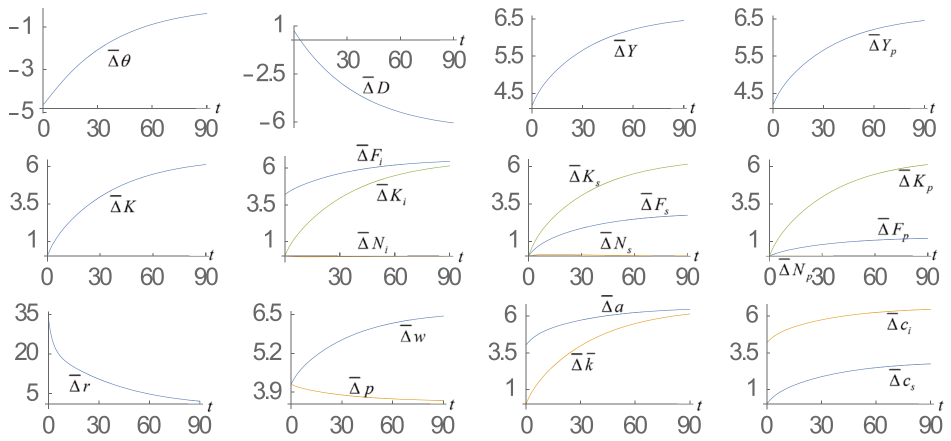

Figure 9. *A Rise in the Total Factor Productivity of the Industrial Sector for the Stable Path*

Impact on the unstable path

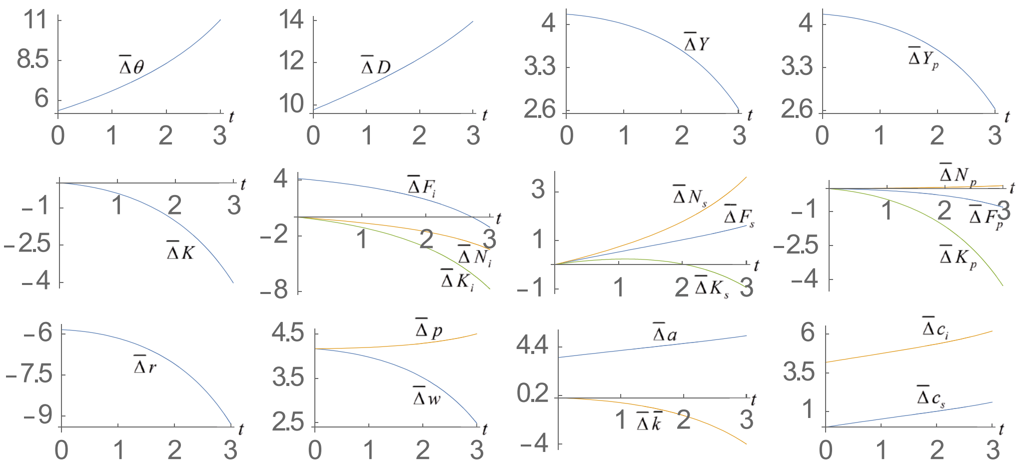

Figure 10. *A Rise in the Total Factor Productivity of the Industrial Sector for the Unstable Path*

The impact on the unstable equilibrium point is listed in (26). The national debt, national output and ratio are increased. The output levels of the three sectors are increased. The consumption levels of industrial good and services are enhanced.

$$\bar{\Delta}\theta = 2.48,\ \bar{\Delta}Y = 7.5,\ \bar{\Delta}D = 10.1,\ \bar{\Delta}K = 8.96,\ \bar{\Delta}Y_p = 7.5,\ \bar{\Delta}N_i = -0.65,\ \bar{\Delta}N_s = 1.1,$$
$$\bar{\Delta}N_p = 0.03,\ \bar{\Delta}K_i = 8.2,\ \bar{\Delta}K_s = 10.1,\ \bar{\Delta}K_p = 8.9,\ \bar{\Delta}F_i = 6.7,\ \bar{\Delta}F_s = 5.1,\ \bar{\Delta}F_p = 1.7,$$
$$\bar{\Delta}w = 7.4,\ \bar{\Delta}r = -1.86,\ \bar{\Delta}p = 3.4,\ \bar{\Delta}a = 9.2,\ \bar{\Delta}\bar{k} = 8.96,\ \bar{\Delta}c_i = 8.65,\ \bar{\Delta}c_s = 5.1. \tag{26}$$

5. Conclusions

This paper developed a neoclassical three-sector economic growth model of endogenous debt with externalities and congestion in a competitive economy. The economy has two production sectors like in the standard two sector neoclassical growth and one public sector. We describe nonlinear dynamic interactions between economic structural change, capital accumulation and public debt under different combinations of taxes on the goods sector, the service sector, the wage income, the rate of interest, consumption of good, and consumption of service. The model has two equilibrium points. The one with higher national output and capital is stable, and the one with lower national output and capital is unstable. We examined how the stable and unstable paths react to changes in different parameters. Although examination between national debts and economic growth are important, there are only a few growth models with dynamics of debts. This study introduced national debts into the neoclassical growth theory. A unique feature of our model is that it introduces various taxes on different parts of economic agents. As our comparative dynamic analysis is limited to a few cases, we can get more insights from further simulation. It is possible to extend the model in some directions. The Solow model and the Uzawa two-sector growth model are most well-known models in the literature of growth theory. Many limitations of our model and possible extensions and generalizations become apparent in the light of the sophistication of the literature.

Appendix: Proving the Lemma

From (2), (4) and (13), we get

$$z \equiv \frac{w}{r_\delta} = \bar{\alpha}_i k_i = \bar{\alpha}_s k_s = \bar{\alpha}_p k_p , \qquad \text{(A1)}$$

where $k_p \equiv K_p / N_p$, $\bar{\alpha}_j \equiv \beta_j / \alpha_j$, $j = i, s, p$. Insert $k_j = z / \bar{\alpha}_j$ and $N_j = K_j / k_j$ in (17)

$$\bar{\alpha}_i K_i + \bar{\alpha}_s K_s + \bar{\alpha}_p K_p = z N . \qquad \text{(A2)}$$

From (A2) and $K_i = K - K_s - K_p$, we have

$$K_p = \alpha_x N z - \alpha_x \bar{\alpha}_i K - (\bar{\alpha}_s - \bar{\alpha}_i)\alpha_x K_s ,$$
$$K_i = -N \alpha_x z + \alpha_x \bar{\alpha}_p K + (\bar{\alpha}_s - \bar{\alpha}_p)\alpha_x K_s , \qquad \text{(A3)}$$

where $\alpha_x \equiv 1/(\bar{\alpha}_p - \bar{\alpha}_i)$. From (8) and (16), we have

$$\gamma \hat{y} N = p F_s .$$

Insert $r_\delta = \alpha_s \bar{\tau}_s p F_s / K_s$ in the above equation

$$\gamma \hat{y} N = \frac{r_\delta K_s}{\alpha_s \bar{\tau}_s} . \qquad \text{(A4)}$$

From (5) and (6) we have

$$\hat{y} = (1 + \bar{\tau}_k r)\bar{k} + \bar{\tau}_w w + (1 + r)d . \qquad \text{(A5)}$$

Insert (A5) and $w = z r_\delta$ in (A4)

$$(\delta_0 + \bar{\tau}_k r_\delta)K + \bar{\tau}_w N z r_\delta + (\delta + r_\delta)D = \frac{r_\delta K_s}{\gamma \alpha_s \bar{\tau}_s}, \qquad \text{(A6)}$$

where we use $r = r_\delta - \delta_k$, $\delta_0 \equiv 1 - \bar{\tau}_k \delta_k$ and $\delta \equiv 1 - \delta_k$. Solve (A6)

$$D = \left(\frac{r_\delta K_s}{\gamma \alpha_s \bar{\tau}_s N} - \delta_0 \bar{k} - \bar{\tau}_k r_\delta \bar{k} - \bar{\tau}_w z r_\delta \right) \frac{N}{\delta + r_\delta} . \qquad \text{(A7)}$$

From (11) and (13) we have

$$r_\delta K_p = \alpha_p \tau (F_i + p F_s) . \qquad \text{(A8)}$$

From (2) and (3) we have

$$r_\delta = \frac{\alpha_i \bar{\tau}_i F_i}{K_i} = \frac{\alpha_s \bar{\tau}_s p F_s}{K_s} . \qquad \text{(A9)}$$

From (A8) and (A9) we have

$$K_p = \alpha_p \tau \left(\frac{K_i}{\alpha_i \bar{\tau}_i} + \frac{K_s}{\alpha_s \bar{\tau}_s} \right) . \qquad \text{(A10)}$$

From (A3) and (A10)

$$\tau_0 K_i = \alpha_x N z - \alpha_x \bar{\alpha}_i K - \left(\alpha_x (\bar{\alpha}_s - \bar{\alpha}_i) + \frac{\alpha_p \tau}{\alpha_s \bar{\tau}_s} \right) K_s , \qquad \text{(A11)}$$

where $\tau_0 \equiv \alpha_p \tau / \alpha_i \bar{\tau}_i$. Insert K_i in (A3) in the above equation

$$K_s = a_z z - a_k \bar{k} , \qquad \text{(A12)}$$

where

$$a_z \equiv (1 + \tau_0) \frac{\alpha_x N}{a_s}, \quad a_k \equiv (\bar{\alpha}_i + \tau_0 \bar{\alpha}_p) \frac{\alpha_x N}{a_s},$$

$$a_s \equiv (\bar{\alpha}_s - \bar{\alpha}_p)\alpha_x \tau_0 + (\bar{\alpha}_s - \bar{\alpha}_i)\alpha_x + \frac{\alpha_p \tau}{\alpha_s \bar{\tau}_s} .$$

Insert (A12) in (A3)

$$K_i = b_z z + b_k \bar{k} . \qquad \text{(A13)}$$

where

$$b_z \equiv (\bar{\alpha}_s - \bar{\alpha}_p)\alpha_x a_z - \alpha_x N , \quad b_k \equiv \alpha_x \bar{\alpha}_p N - (\bar{\alpha}_s - \bar{\alpha}_p)\alpha_x a_k .$$

Insert (A12) and (A13) in (A10)

$$K_p = \beta_z z + \beta_k \bar{k} , \qquad \text{(A14)}$$

where

$$\beta_z \equiv \alpha_p \tau \left(\frac{b_z}{\alpha_i \bar{\tau}_i} + \frac{a_z}{\alpha_s \bar{\tau}_s} \right), \quad \beta_k \equiv \alpha_p \tau \left(\frac{b_k}{\alpha_i \bar{\tau}_i} - \frac{a_k}{\alpha_s \bar{\tau}_s} \right).$$

From (2) we have

$$r_\delta = \alpha_i \bar{\tau}_i A_i A_p^{\theta_{pi}} K_p^{\theta_{pi} \alpha_{0p}} N_p^{\theta_{pi} \beta_{0p}} K_i^{\theta_{ei}} \left(\frac{K_p}{K_i} \right)^{\theta_{ci}} k_i^{-\beta_i} , \qquad \text{(A15)}$$

where we also use (10). Insert $k_i = z / \bar{\alpha}_i$ and $N_p = \bar{\alpha}_p K_p / z$ in (A15)

$$r_\delta = \Lambda(z, \bar{k}) \equiv \alpha_i \bar{\tau}_i A_i A_p^{\theta_{pi}} K_p^{\theta_{pi} \alpha_{0p}} K_i^{\theta_{ei}} \left(\frac{\bar{\alpha}_p K_p}{z} \right)^{\theta_{pi} \beta_{0p}} \left(\frac{K_p}{K_i} \right)^{\theta_{ci}} \left(\frac{\bar{\alpha}_i}{z} \right)^{\beta_i} . \qquad \text{(A16)}$$

From (A12)-(A14) and (A16), we know that r_δ is a function of z and $\bar{k}$. We don't explicitly give the presentation as it is too tedious. Insert (A16) and (A12) in (A7)

$$D = \Delta(z, \bar{k}) \equiv \left(\frac{r_\delta K_s}{\gamma \alpha_s \bar{\tau}_s N} - \delta_0 \bar{k} - \bar{\tau}_k r_\delta \bar{k} - \bar{\tau}_w z r_\delta \right) \frac{N}{\delta + r_\delta}. \quad \text{(A17)}$$

By the following procedure we can determine all the variables as functions of z and $\bar{k}$: D by (A17) → r_δ by (A16) → $r = r_\delta - \delta_k$ → K_s by (A12) → K_i by (A13) → K_p by (A14) → $k_j = z/\bar{\alpha}_j$, $j = i, s, p$ → $N_j = K_j / k_j$ → K_p by (A15) → K_i by (A10) → K_s by (A6) → $N_j = K_j / k_j$, $j = i, s, p$ by (A15) → $w = z r_\delta$ → Ω_i and Ω_s by the specified forms → F_j by the specified forms → p by (A9) → $\hat{y} = p F_s / \gamma N$ → $\underline{c}_s, c_i$ and s by (8) → T_p by (14) → $K = \bar{k} N$ → Y_p by (10) → $a = \bar{k} + D/N$. By (9), (15) and this procedure, we have

$$\dot{D} = \Psi(z, \bar{k}) \equiv r D + Y_p - T_p, \quad \text{(A18)}$$

$$\dot{\bar{k}} = \tilde{\Psi}(z, \bar{k}) \equiv s - \bar{k} - \frac{D}{N} - \frac{\Psi}{N}. \quad \text{(A19)}$$

Taking derivatives of (A15) with respect to time yields

$$\dot{D} = \frac{\partial \Delta}{\partial z} \dot{z} + \frac{\partial \Delta}{\partial \bar{k}} \dot{\bar{k}}. \quad \text{(A20)}$$

From (A18) and (A20), we solve

$$\dot{z} = \hat{\Psi}(z, \bar{k}) \equiv \left(\Psi - \frac{\partial \Delta}{\partial \bar{k}} \tilde{\Psi} \right) \left(\frac{\partial \Delta}{\partial z} \right)^{-1}, \quad \text{(A21)}$$

where we also use (A19). We thus proved the Lemma.

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