About the Measures of Skewness and Kurtosis

Assoc. Prof. Todor Kaloyanov, Ph.D.

Summary: This article sets forth a comparative analysis of five coefficients measuring the degree of skewness in empirical statistic distributions. The coefficients are calculated for the distributions of live-births as per the age of the mother. The data are in total for Bulgaria, respectively for all children born, for first and second child during the period from 1961 to 2008. A discussion is presented as regards the cognitive meaning and reasons for variation in their values. The link between skewness and kurtosis is being examined and the necessity of their joint use is being justified given, the existence of empirical distributions that are not subject to the law of Laplace – Gauss. Unlike the predominant practice to place the focus of attention mostly on kurtosis in symmetrical distributions, the opposite task is set here – to analyze the existence of skewness given a different degree of kurtosis.

Key words: statistical distribution, skewness coefficient, excess kurtosis coefficient.

JEL: C10, C16, C46.

Traditionally, in the presentation of summarizing numerical characteristics of distributions in literature it is pointed out that skewness characterizes the sideward skewing of distribution. It is noted that sideward skewing may be left and right, and accordingly skewness is negative or positive [Venetskiy, Venetskaya (1979; 16), Mansfield (1987; 44)]. Unlike kurtosis for the cognitive meaning of which a discussion is under way, the issue of the meaning of skewness is broached comparatively more seldom [Groeneveld, R.A. and Meeden, G. (1984; 391)]. Very often in practical research it is accepted that the measure constructed on the grounds of moments is to be preferred, after which the line is drawn.

Each of the characteristics of statistical distribution has a certain cognitive meaning. It reflects in its own manner the influences as a result of which have formed the respective meanings of the sign, inherent to the units through which the researched phenomenon is expressed. Depending on the problem being solved and the type of empirical distribution, the different characteristics are also used. The focus of attention in this article is oriented towards skewness and some of its measures.

In the analysis of empirical asymmetric distributions two questions are posed as a rule:

- First, what is the type of skewness – positive or negative;
- Second, what is the degree of skewness – comparatively big or comparatively small, i.e., to measure the degree of deviation from symmetrical distribution;

Behind these questions a third one is hidden – as a result of which influences asymmetric distribution has been obtained? In most of the cases this question is not asked. Moreover, to skewness, and as a rule to kurtosis, no attention is given to statistical practice. But what happens when these questions are asked in a research of empirical distributions[1]?

The answer to the first questions seems too simple. The frequency polygon of empirical distribution is constructed and if the left tail of the curve is skewed, the distribution is with a left, negative skewness. And vice versa, when the distribution is with a steep left tail and skewed, slanting right tail, a right, positive skewness is present.

The answer to the second question is given with the respective numerical measure. But which one will it be, provided there are different possible measures, such as the coefficients of Pearson, Bowley, Kelly, a coefficient based on the third moment[2]. Each one of these has a different construction, some of these have borderline values, and others do not have such. Which one of these is appropriate, which one of these may be trusted?

Depending on their construction, measures can be divided into two types:

- measures of a "pure" type with three varieties. The first variety is the measure based on a median in position – first, second and third quartile. As it is well known, the skewness coefficient known as Bowley coefficient accounts only frequencies in an apparent type. In determining its value, the meanings of the sign do not participate directly.

$$A_B = \frac{Q_1 + Q_3 - 2M_e}{Q_3 - Q_1}$$

where Q_1 and Q_3 are accordingly first and third quartile M_e is the median.

The second "pure" measure is the moment coefficient of skewness. In its construction take part the first initial, the second and third central moments. Its value is formed with the concurrent participation of the meanings of the respective signs and frequencies.

$$A_M = \frac{\dfrac{\sum_{i=1}^{k}(x_i - \mu)^3 f_i}{\sum_{i=1}^{k} f_i}}{\left(\sqrt{\dfrac{\sum_{i=1}^{k}(x_i - \mu)^2 f_i}{\sum_{i=1}^{k} f_i}}\right)^3}$$

The third measure proposed by Vazharov[3], is based on three values obtained as per the formula of arithmetic mean and appears as follows:

$$K_a = \frac{\mu_1 + \mu_2 - 2\mu}{\mu_2 - \mu_1},$$

1 Unimodal statistical distributions are meant here.
2 It is known that with the aid of odd moments of an order higher than third skewness is also characterized.
3 This measure has not been published till now.

where μ is the arithmetic mean for all units of the aggregate being studied;
μ_1 is the arithmetic mean for the units having values lower than the median of the entire aggregate μ;
μ_2 is the arithmetic mean for the units having values higher than the median of the entire aggregate μ.

This coefficient is compatible with values from -1 to +1.

To this type also refer the Kelly coefficient using deciles and percentiles.

- measures of "mixed" type. Such are the measures of Pearson. In their construction takes part not only the first initial and second central moment, but also a second central moment and also a density mean (mode) and a mean of position (median) .

$$A_{P1} = \frac{\mu - M_0}{\sigma} \text{ and } A_{P2} = \frac{3(\mu - M_e)}{\sigma},$$

Where the symbols are known.

As an illustration of the difficulties faced by a researcher in the choice of skewness measure, three examples will be examined. They are within the field of demographic statistics and refer to the distributions of live-births as per the mother's age. The data are in total for Bulgaria, respectively for all children born, for first and for second child during the period from 1961 to 2008. For each distribution are calculated the arithmetic mean, mode, median, first and third quartile, mean quadratic deviation, skewness

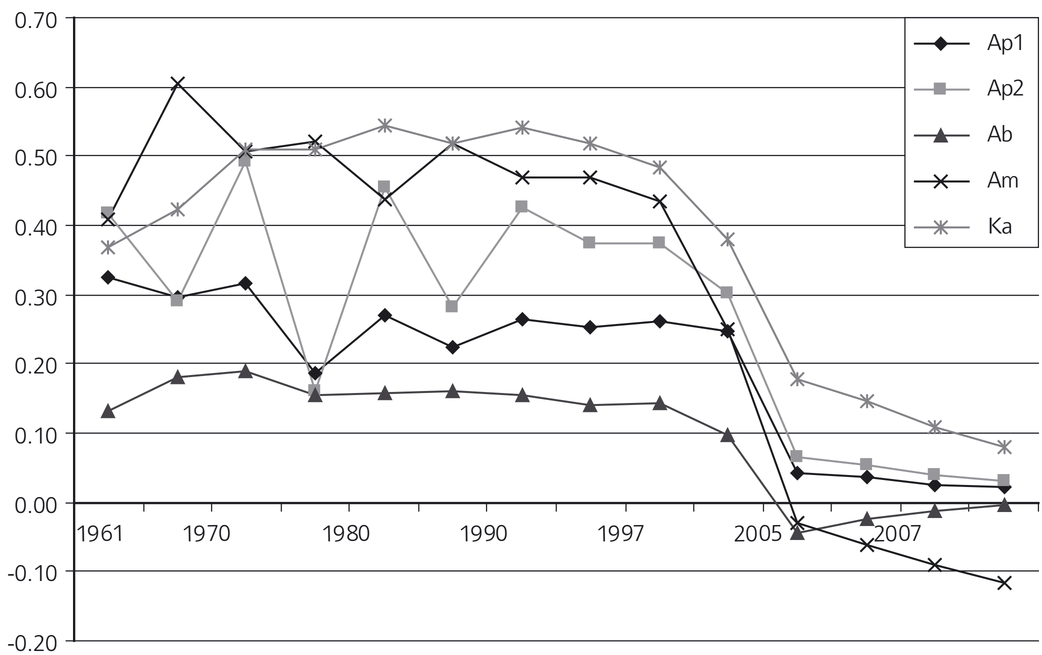

Figure 1. The values of the coefficients of skewness of distributions for all children in the Republic of Bulgaria during the period 1961-2008.

TTable 1. Distribution and distribution parameters of all live-born children in the Republic of Bulgaria in the period 1961-2008

Age	Years													
	1961	1965	1970	1975	1980	1985	1990	1995	1997	2000	2005	2006	2007	2008
10-14	126	167	213	238	262	407	502	466	397	417	399	383	407	456
15-19	21577	23266	23599	23649	24301	22804	22015	15812	12674	12370	9679	9861	9673	9675
20-24	52506	50653	64198	65281	59437	52976	46872	29872	26242	27237	20628	20716	20147	20312
25-29	38696	29996	31956	38761	30029	28282	23179	17007	16343	21577	22871	23428	23427	23479
30-34	16885	14663	12854	12063	10727	10752	8954	6163	5904	8851	13113	14627	15970	17436
35-39	6412	5437	4765	3646	2733	3180	3027	2171	2062	2604	3796	4355	4977	5516
40-44	1376	1429	1029	914	595	521	603	452	479	543	550	571	699	792
45-49	199	180	131	116	106	24	20	22	24	27	18	25	31	35
Totally	137777	125791	138745	144668	128190	118946	105172	71965	64125	73626	71054	73966	75331	77701
μ	25.27	24.40	24.36	23.97	24.09	23.91	23.96	23.96	24.29	24.96	26.18	26.39	26.67	26.85
M_0	23.46	22.85	22.79	23.05	22.72	22.75	22.56	22.61	22.89	23.62	25.93	26.18	26.53	26.72
M_e	24.49	23.90	23.55	23.71	23.33	23.42	23.21	23.30	23.62	24.41	26.05	26.29	26.59	26.79
Q_1	21.21	20.79	20.85	20.94	20.63	20.62	20.40	20.29	20.56	21.03	21.86	21.99	22.17	22.29
Q_3	28.76	28.38	27.51	27.49	27.02	27.30	27.05	27.30	27.69	28.52	29.89	30.37	30.89	31.25
σ	5.59	5.23	4.98	4.94	5.07	5.18	5.31	5.31	5.35	5.43	5.70	5.76	5.87	5.94
μ_1	21.03	20.91	21.14	21.15	21.02	20.95	20.84	20.69	20.79	20.85	20.79	20.78	20.77	20.76
μ_2	34.46	34.53	34.42	34.24	34.00	33.98	34.20	34.28	34.33	34.07	33.92	33.92	33.99	34.01
A_{P1}	0.32	0.30	0.32	0.19	0.27	0.22	0.26	0.25	0.26	0.25	0.04	0.04	0.02	0.02
A_{P2}	0.42	0.29	0.49	0.16	0.45	0.28	0.43	0.37	0.37	0.30	0.07	0.05	0.04	0.03
A_B	0.13	0.18	0.19	0.15	0.16	0.16	0.16	0.14	0.14	0.10	-0.04	-0.02	-0.01	0.00
A_M	0.41	0.60	0.51	0.52	0.44	0.52	0.47	0.47	0.43	0.25	-0.03	-0.06	-0.09	-0.12
K_A	0.37	0.42	0.51	0.51	0.55	0.52	0.54	0.52	0.48	0.38	0.18	0.15	0.11	0.08

coefficients of Pearson in two varieties – of Bowley and the moment one and the coefficient is constructed by Hr. Vazharov.

The data shown in Table 1 and the diagram in Figure 1 that is constructed on the base of these confirm the expectations of different values of the coefficients. This is completely logical since all five measures are constructed on a different base. The different properties of the elements taking part in the construction of individual measures, as well as their different sensitivity to changes in frequencies are reflected in the coefficient values.

As shown in Figure 1 all five measurements show one and the same trend in the alteration of the degree of skewness of the researched distributions. This trend is one of decrease. At the same time some differences are present. Bowley's coefficient has the lowest values till the year 2005, with values falling within the range between -1 to +1. In this case the interest is oriented to the manner of alteration of skewness coefficients.

Relatively lowest variations are shown by the coefficient of Bowley which is due to its limits. The most abrupt and biggest are the variations in Pearson's coefficient till the year 1990, which is based on the difference between the arithmetic mean and mode.

The coefficient K_a, using the differences between the values of the arithmetic mean is the biggest in all years. On its part, the moment

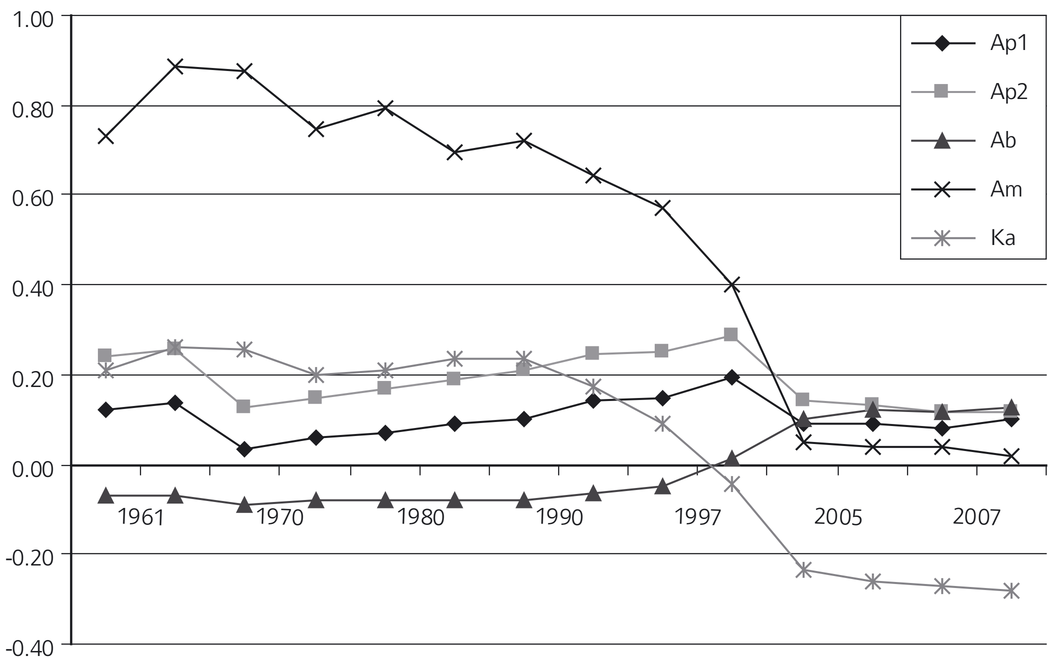

Figure 2. The values of the coefficients of skewness of the distribution for a first child in the Republic of Bulgaria during the period 1961-2008.

Table 2. Distribution and distribution parameters of first live-born child in the Republic of Bulgaria in the period 1961-2008

Age	Years													
	1961	1965	1970	1975	1980	1985	1990	1995	1997	2000	2005	2006	2007	2008
10-14	122	163	211	232	255	394	490	452	386	410	386	374	399	446
15-19	18797	20273	19705	19111	19700	18163	17288	12404	10295	9993	7624	7594	7399	7462
20-24	28657	27631	34968	33250	31280	26880	25573	18100	17199	17947	13712	13835	13179	13482
25-29	7916	6764	6329	8108	7506	6728	6561	6110	7045	10462	12881	13437	13049	13297
30-34	2214	2039	1663	1652	1914	1833	1878	1598	1667	2947	5361	6302	6630	7044
35-39	747	783	590	553	437	537	585	560	568	866	1314	1594	1661	1792
40-44	96	144	152	123	97	96	133	107	137	186	159	212	269	270
45-49	5	10	17	30	62	6	2	6	9	16	6	7	15	12
Totally	58554	57807	63635	63059	61251	54637	52510	39337	37306	42827	41443	43355	42601	43805
μ	22.15	21.91	21.87	22.04	21.94	21.90	21.96	22.26	22.72	23.55	24.89	25.19	25.35	25.42
M_0	21.61	21.30	21.74	21.80	21.64	21.51	21.52	21.61	22.02	22.58	24.40	24.70	24.89	24.85
M_e	21.81	21.53	21.70	21.83	21.71	21.63	21.66	21.88	22.32	23.07	24.64	24.95	25.12	25.19
M_0	0.24	0.26	0.13	0.15	0.17	0.19	0.21	0.25	0.25	0.28	0.14	0.13	0.12	0.12
M_e	-0.07	-0.07	-0.09	-0.08	-0.08	-0.08	-0.08	-0.07	-0.05	0.01	0.10	0.12	0.12	0.13
Q_1	0.73	0.89	0.88	0.74	0.79	0.69	0.72	0.64	0.57	0.40	0.05	0.04	0.04	0.02
Q_3	1.34	1.84	2.82	2.29	2.43	1.56	1.61	1.20	1.10	0.50	-0.28	-0.33	-0.32	-0.36
σ	4.37	4.42	4.03	4.11	4.18	4.28	4.40	4.64	4.73	5.05	5.43	5.55	5.67	5.70
μ_1	17.47	17.46	17.45	17.44	17.44	17.39	17.36	17.32	17.32	17.30	17.26	17.27	17.24	17.22
μ_2	29.31	29.54	29.33	28.92	28.87	29.22	29.38	29.29	29.18	29.28	29.64	29.84	29.97	30.04
A_{P1}	0.12	0.14	0.03	0.06	0.07	0.09	0.10	0.14	0.15	0.19	0.09	0.09	0.08	0.10
A_{P2}	0.24	0.26	0.13	0.15	0.17	0.19	0.21	0.25	0.25	0.28	0.14	0.13	0.12	0.12
A_B	-0.07	-0.07	-0.09	-0.08	-0.08	-0.08	-0.08	-0.07	-0.05	0.01	0.10	0.12	0.12	0.13
A_M	0.73	0.89	0.88	0.74	0.79	0.69	0.72	0.64	0.57	0.40	0.05	0.04	0.04	0.02
K_A	0.21	0.26	0.26	0.20	0.21	0.24	0.23	0.17	0.09	-0.04	-0.23	-0.26	-0.27	-0.28

coefficient is the only measure demonstrating increase in negative skewness after the year 2005, while Bowley's coefficient changes in the opposite direction. As per the latter, distribution becomes almost symmetrical. The changes in the values of Pearson measures rather approximate those of the moment coefficient, but as per the latter negative skewness is not existent yet.

As it may be established from Table 2 and Figure 2 – according to Bowley's coefficient, distributions till the year 1997 have negative skewness which is almost identical in size. As per the measure K_A in the year 2000 distributions already have negative skewness. At the same time, the other three measures evidence that not any of the distributions being analyzed has negative skewness. The moment coefficient shows that after the year 1965 skewness is continuously on the decrease and distributions become almost symmetrical. The two Pearson's coefficients change in parallel and there are no "disagreements" between them. An important point in this case is that after the year 2005 the values of all four measures are close, while preserving the differences in the orientation of alteration in skewness – Table 2. On the whole, the changes in the skewness of distributions are relatively smooth and within a narrow range. Larger are the alterations in the values of the moment coefficient and of K_A.

Much more different is the picture in Figure 3. In the alteration in the values of all coefficients an alternation is established as regards the skewness increase, drop, new

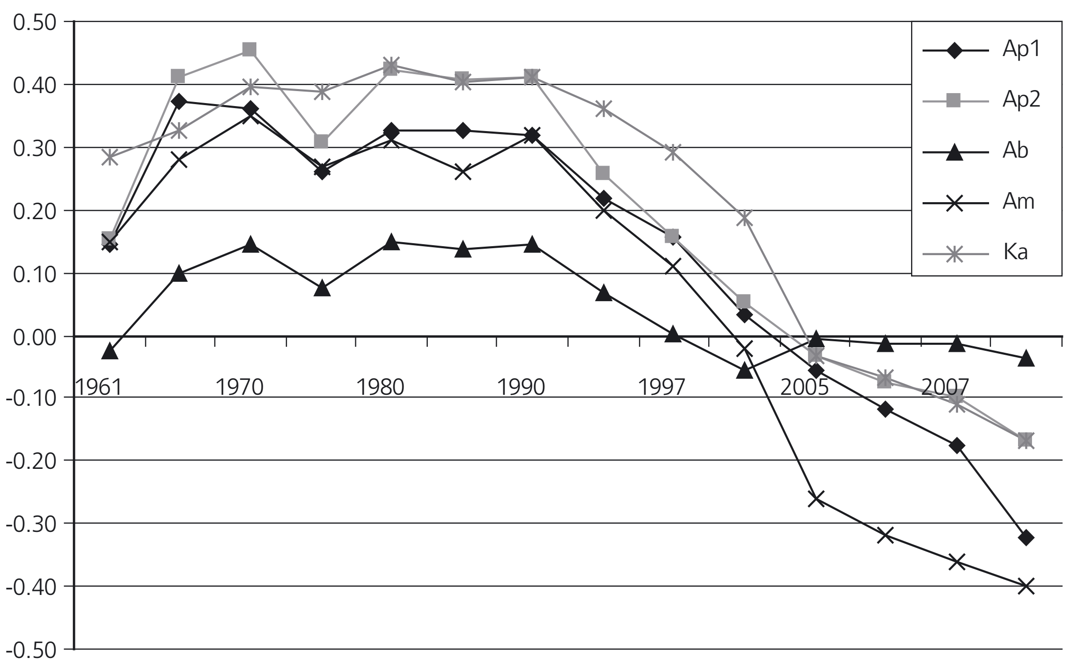

Figure 3. The values of the coefficients of distributions skewness for a second child in the Republic of Bulgaria during the period 1961-2008.

Table 3. Distribution and distribution parameters of second live-born child in the Republic of Bulgaria in the period 1961-2008

Age	Years													
	1961	1965	1970	1975	1980	1985	1990	1995	1997	2000	2005	2006	2007	2008
10-14	4	4	2	6	7	13	12	14	10	7	13	8	8	10
15-19	2517	2736	3408	4031	4015	4025	4053	2974	2086	2089	1743	1960	1891	1837
20-24	19187	18403	23102	26099	22633	20995	17186	9369	7049	7012	4992	4864	4912	4784
25-29	20508	15248	16883	22482	15796	15371	12156	8163	7006	8310	7537	7498	7706	7632
30-34	6617	5916	5705	6075	5035	5194	4261	2833	2724	3946	5765	6354	7137	8115
35-39	1413	1327	1397	1301	935	1153	1132	742	645	814	1465	1737	2118	2528
40-44	119	197	162	210	127	125	145	112	89	125	151	154	183	258
45-49	12	8	3	10	16	1	5	5	5	3	8	3	4	12
Totally	50377	43839	50662	60214	48564	46877	38950	24212	19614	22306	21674	22578	23959	25176
μ	26.07	25.82	25.43	25.44	25.10	25.24	25.14	25.29	25.73	26.32	27.65	27.83	28.17	28.59
M_0	25.43	24.16	23.80	24.30	23.66	23.76	23.62	24.21	24.96	26.15	27.95	28.49	29.15	30.40
M_e	25.85	25.21	24.74	24.99	24.48	24.62	24.48	24.87	25.47	26.23	27.71	27.97	28.35	28.90
Q_1	22.63	22.23	22.00	22.11	21.79	21.83	21.65	21.64	21.99	22.48	23.67	23.78	24.16	24.65
Q_3	28.92	28.85	28.40	28.34	28.09	28.29	28.27	28.55	28.97	29.59	31.71	32.05	32.42	32.85
σ	4.41	4.46	4.52	4.37	4.43	4.56	4.77	4.96	4.97	5.05	5.41	5.53	5.56	5.64
μ_1	21.92	21.85	21.86	21.83	21.74	21.69	21.54	21.29	21.35	21.35	21.19	21.05	21.10	21.10
μ_2	33.53	33.67	33.69	33.65	33.51	33.59	33.80	33.83	33.71	33.60	33.71	33.75	33.82	33.91
A_{P1}	0.14	0.37	0.36	0.26	0.33	0.33	0.32	0.22	0.16	0.03	-0.06	-0.12	-0.18	-0.32
A_{P2}	0.15	0.41	0.46	0.31	0.42	0.41	0.41	0.26	0.16	0.05	-0.03	-0.08	-0.10	-0.17
A_B	-0.02	0.10	0.14	0.07	0.15	0.14	0.14	0.07	0.00	-0.06	-0.01	-0.01	-0.02	-0.04
A_M	0.73	0.89	0.88	0.74	0.79	0.69	0.72	0.64	0.57	0.40	0.05	0.04	0.04	0.02
K_A	0.28	0.33	0.40	0.39	0.43	0.40	0.41	0.36	0.29	0.19	-0.03	-0.07	-0.11	-0.17

increase, preservation of the level reached, lowering and changing to negative skewness. After the year 2000 the coefficient of Bowley evidences the preservation of the reached level of negative skewness, while as per the rest of the measures negative skewness is on the increase. The values of all five measures show one-directional alterations in time. For the first time values of the moment coefficient till the year 2000 are between those of the other coefficients – in the case of Pearson's and Bowley's coefficients. For a short span a similar fact is found in the distribution of all children born for two years – 2000 and 2005.

The problem arising in the choice of skewness measure is obvious. Each of the reviewed coefficients measures in its own manner the deviation from symmetrical distribution. This means that also in the interpretation of the obtained values an account should be rendered as to their cognitive meaning.

The first coefficient of Pearson is based on the difference between the arithmetic mean and mode. Therefore, it measures the difference between one quantity that is a function of two elements – the values of the sign in the individual units and frequencies, and a second one that is a function of the distribution of frequencies in three neighboring meanings or three neighboring ranges. Similar is the notion with the second measure of Pearson as well.

With Bowley's coefficient the distribution of frequencies in six intervals is taken into consideration based on which quartiles are calculated.

Taking into account that:

- Firstly, arithmetic mean is a certain centre of statistical distribution which, with asymmetrical distributions does not coincide with the maximum concentration of the units, but depends on the degree of concentration and vice versa – on the degree of deconcentration, and
- Secondly, the mode is the meaning having the greatest concentration of units,

then the first Pearson's coefficient measures skewness related mainly to the concentration of units which are contained between the point of biggest concentration and the distribution centre, determined through the arithmetic mean.

The second coefficient of Pearson measures mainly the degree of skewness that is related to the concentration of units between the median and the arithmetic mean. Taking into account that in the general case these two means are with closer values, it may be accepted that it measures the degree of concentration within a narrower range.

The coefficient of Bowley may be reviewed in an analogical manner. It is typical for it

Table 4. Moment excess kurtosis for all children born, for first and second child in the Republic of Bulgaria during the period 1961-2008

Years	1961	1965	1970	1975	1980	1985	1990	1995	1997	2000	2005	2006	2007	2008
E-total	0.12	0.62	0.74	0.69	0.30	0.43	0.33	0.33	0.32	0.01	-0.38	-0.44	-0.47	-0.50
E-first	1.34	1.84	2.82	2.29	2.43	1.56	1.61	1.20	1.10	0.50	-0.28	-0.33	-0.32	-0.36
E-second	0.19	0.16	0.26	0.51	0.41	0.17	0.21	0.09	-0.02	-0.16	-0.36	-0.45	-0.44	-0.39

to measure skewness that is related to the concentration of units in six ranges, but very often the ranges are only three. A specific variety of the coefficient is also K_A. With it however, the meanings of the sign are accounted in apparent manner and the respective frequencies both for the distribution as a whole and for the parts thereof, divided by its arithmetic mean.

The moment coefficient measures the degree of skewness which is related not only to the concentration of units in one or two ranges, but also accounts in apparent manner the presence of units in all ranges. In other words, it renders account also of the units deconcentration as per the meanings of the sign for which the particular distribution has been formed. Thus, not only the raising of differences to third power is a reason for the higher values of the moment coefficient, but the degree of deconcentration has its contribution as well[4].

Grounds for such a method of interpretation of measures may be found in the three examples and in the following Table 4 and Figure 4, in which are presented the values of the moment excess kurtosis.

According to the data in Table 4 and Figure 4 until 2005 the concentration of the units in relation to the arithmetic mean is the biggest as regards first child distributions. As evident in Figure 2, for this order the values of the moment coefficient differ the most from those with the remaining three measures given the highest excess, that is, given the biggest concentration of units. With the decrease in the degree of concentration to

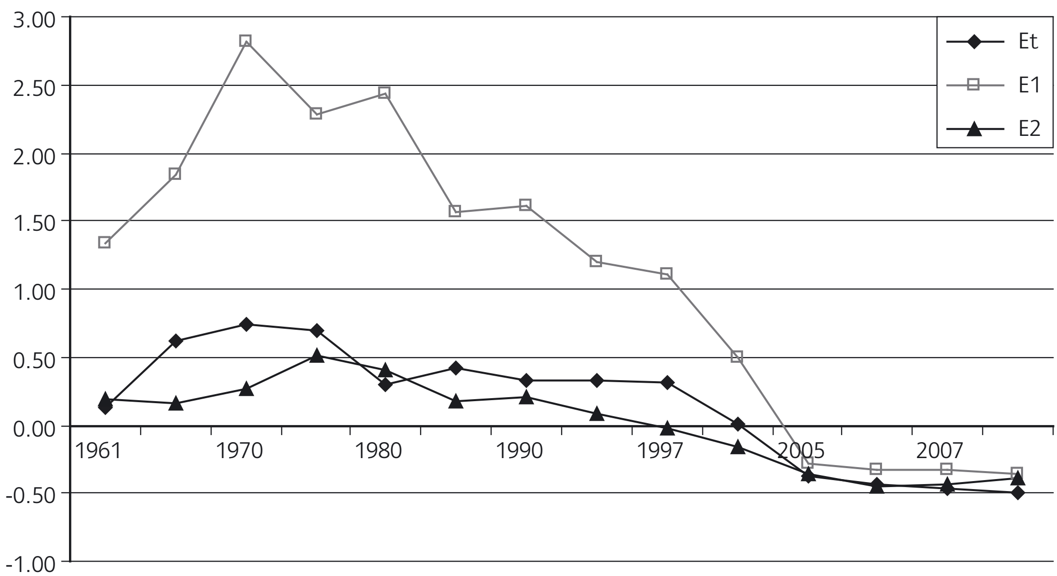

Figure 4. Moment kurtosis for all children born, for a first child and for a second child in the Republic of Bulgaria during the period 1961-2008.

4 In the cases where data are not presented in a range statistical order, only the method of calculation of median and mode will change, without altering the essence of skewness and excess measures.

a certain extent, the differences between the coefficients are also reduced. This is established as regards the distributions for all children born and for first child – Figure 1 and Figure 2. Rather different is the situation with the second child distributions. Characteristic for the latter is that the as per the excess kurtosis coefficient, the concentration degree is lesser than that for first child and all children. Besides, the values of the moment skewness coefficient till 2000 are between those of the coefficients of Pearson and Bowley. After the year 2000 when the degree of over-deconcentration in second child distribution increases, the difference between the moment skewness coefficient and the other coefficients increases as well. What is important in this case is that all parameters indicate an intensification of negative skewness.

In literature the main discussion issue is about the meaning of excess kurtosis coefficient, at that mainly in symmetrical distributions, starting with K. Pearson, Student till nowadays. In the examples reviewed herein the opposite situation is being analyzed. What is the sense of skewness in distributions with various excess? It can be seen that both characteristics of the distribution form are related to the degree of concentration of the units within the particular distribution. However, skewness is also related to the location of concentration and also depending on the type of measure, i.e., of its construction, and different degree of concentration is measured as well.

The skewness and excess measures which are constructed on the base of moments have the capacity to account the concentration degree in relation to one centre – the arithmetic mean. What is different between these two measures is that the coefficient of skewness also shows the existence of any deviations typical of a certain part of the units. Thus, a tribute is paid to any specific impacts that have conditioned a different behaviour for a part of the units.

The discussion on determining the most appropriate skewness and excess measures is still under way. Each of the existing measures has its pros and cons. The choice of a specific measure depends on the type of data and on the particular problems to be solved. To date, a clear and acceptable systematization of various measures to be of aid to the researchers in their practical activities is still not available.

Neglecting the existence of skewness and excess in empirical distributions rather frequently leads to the adoption of incorrect decisions. In other cases the use and considering the existence of skewness and excess allows to find a solution to problems that have been assumed as almost irresolvable over long periods of time[5].

Bibliography

1. Abramowitz, Milton; Stegun, Irene A., eds. (1972). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, New York: Dover Publications, ISBN 978-0-486-61272-0.

2. Bowley, A. L. (1920). Elements of Statistics, New York: Charles Scribner's Sons.

3. Венецкий, И. Г., В. И. Венецкая (1979). Основные математико-статистические

[5] On this issue, see Kaloyanov, T. A Study of Dynamics and Fertility of Women in Bulgaria through comparing distributions 1961-2008. UPH "Economy", UNWE, Sofia, 2011, pp. 30-45.

понятия и формулы в экономическом анализе, Москва.

4. Chissom, Brad S. (1970). "Interpretation of the Kurtosis Statistics", The American Statistician, Vol. 34, No. 4, pp. 19-22.

5. Cramer, H. (1945). "Mathematical Methods of Statistics", 1, N.J.: Princeton University Press.

6. Darlington, R. B. (1970). "Is Kurtosis Really Peakedness?" The American Statistician, 24, pp. 19-22.

7. Dodge, Y. and Rousson, V. (1999). "The Complications of the Fourth Central Moment." The American Statistician, 53, pp. 267-269.

8. Dyson, F. J. (1943). "A Note on Kurtosis" Journal of the Royal Statistical Society, 106, 4, 360-361.

9. Finucan, H. M. (1964). "A Note on Kurtosis," Journal of the Royal Statistical Society. Series B, 26, 1, pp. 111-112.

10. Groeneveld, R. A., and Meeden, G. (1984). "Measuring Skewness and Kurtosis", The Statistician, 33, pp. 391-399.

11. Hildebrand, David K. (1971). "Kurtosis Measures Bimodality?", The American Statistician, 25, No. 1, pp. 42-43.

12. Kendall, M. G. and Stuart, A. (1969). The Advanced Theory of Statistics (Vol. 1). London: Charles W. Griffin.

13. Kotz, Samuel and Seier, Edith (2007). "An analysis of quantile measures of kurtosis: center and tails", Journal Statistical Papers, Springer Berlin/ Heidelberg.

14. MacGillivray, H. L. (1986). "Skewness and Skewness: Measures and Orderings", The Annals of Statistics, 14, pp. 994-1011.

15. MacGillivray, H. L., Balanda, K. P. (1988). "The relationships between skewness and kurtosis", Austral. J. Statist. 30 (3). pp. 319-337.

16. Mansfield, E. (1987). Statistics for Business and Economics. Methods and Applications. Third ed. W.W. Norton & Company, Inc. New York.

17. Pearson, K. (1895). Contributions to the mathematical theory of evolution, II: Skew variation in homogeneous material. Philosophical Transactions of the Royal Society of London, 186, pp. 343-414.

18. Pearson, K. (1905). "Skew variation, a rejoinder". Biometrika, 4, pp. 169-212.

19. Student, (July 1927). "Errors of Routine Analysis" Biometrika, 19: pp. 151-164.