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Examining the Role of Structural Change in a Phillips Curve: Bivariate GARCH DCC Analysis

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Abstract

The aim of this study is to empirically test the stability of the traditional Phillips curve using a bivariate GARCH-DCC model. We tried to establish the dynamic correlation between consumer price index inflation and unemployment rate by considering the autoregressive conditional heteroskedasticity (ARCH) effect. Eventually, we analysed the Tunisian data spanning over the first quarter of 1974 up to the first quarter of 2016. The results revealed that the correlation sign between CPI inflation and unemployment changed from negative to positive during recession periods. Our results suggest two main implications for policymakers. First, monetary policy may have more important and long-lasting effects on unemployment during a recession period compared to a boom one. Second, since keeping the unemployment rate at a reasonable level is a priority for the monetary authorities, the CPI inflation-unemployment nexus has to be evaluated before implementing any policy, especially when the economy is strong.

Keywords: Phillips curve, Structural breaks, GARCH-DCC model, Non-stationary data.

JEL: E24; E31; C51.

1. Introduction

The form and the stability of the Phillips curve have attracted much attention in macroeconomics because of its decisive implications for monetary policy. This curve and its presumed options were revealed to be illusionary. During the 1960s the decline of unemployment was accompanied by harsh inflation. The following decade, the 1970s, witnessed an inverse relationship between the two variables and many countries experienced an unemployment growth accompanied by a simultaneous worsening inflation. Such a phenomenon was called stagflation. However, the late eighties were the stage for a simultaneous decrease of both inflation and unemployment. Such observations made it essential to revisit the idea of trade-off between inflation and unemployment.

Most of the empirical studies related to this issue show that the Phillips curve was reinvestigated because of economic and environmental changes such as the perturbations affecting the global offer. These can be summarized as follows: the petroleum

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shocks of 1973-74 and 1978-79; the great flow of new workers on the labour market during the same periods leading to a swift increase in unemployment with unchanging wage levels; the technological changes; the admission conditions to unemployment insurance and the lingering slow-down of labour productivity gains.

Taking into account the progress in econometrics, many recent studies have succeeded in estimating a new Phillips curve. In their analysis of inflation-unemployment relationship, they integrated some diverse determinants such as the anticipated inflation, real marginal cost, productivity and downward nominal wage rigidity. Their Phillips curve estimations were revealed to be stable at least in the short run (Galí and Gertler, 1999; Mankiw and Reis, 2001; Gali et al. 2005; Karolina, 2006; Furuoka, 2007; Karanassou and Sala, 2008, 2010).

Opposite to the above studies and others, the novelty of this research stems from the fact that rather than focusing on affirming or negating the existence of the Phillips curve, this paper investigated the dynamic correlation between consumer price index (CPI) inflation and unemployment rate. The second bonus in the paper's added value is that, methodologically speaking, it adopted the autoregressive conditional heteroskedasticity (ARCH) effect. Russell and Chowdhury's (2013) recent study is another notable addition to the body of knowledge on this topic. The authors retrieved the standard empirical results of the Phillips curves based on a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model with structural breaks. Rather than focusing on the stable relationship between CPI inflation and unemployment rate in the long run, the present paper focused on the time-varying correlations in this relation by considering the time series properties such as serial correlations, the ARCH process, and fat tails. The presence of these properties presumably might have prevented the traditional econometric models from accurately describing the Phillips curve. Therefore, we used the GARCH-DCC (dynamic conditional correlation) model (Rahman and Serletis, 2012; Jones and Olson, 2013) to investigate the stability of the traditional Phillips curve.

Indeed, it is generally accepted that the correlation between CPI inflation and unemployment rate increases during recession periods; this has motivated our analysis of GARCH-DCC as an estimation model during periods of structural breaks – periods during which we know that the correlation dynamics is most likely to change significantly. We examined several periods, which were immediately followed by an acceleration of either CPI inflation or unemployment rate. During the liberalisation period and what became known as the amplification of syndicalism (1971-1985), the inflation rate reached 14.85% in 1982Q3 and unemployment attained 18.5% in 1971[1]. The 1986–1995 period was for the application of the structural adjustment programme: This was a transition period marked by an unstable inflation rate that still ranged between 8.9% and 4% while unemployment was fixed at around 14%. This was followed by the period of the emergence of a new commercial world order coinciding with the birth of the World Trade Organization (WTO) in 1995. The inflation rate in Tunisia was maintained at a comparable level of that of its European economic partners declining remarkably from 6.51% in 1995Q3 to 2.77% in 2007Q2. As for the unemployment rate, it went from 14.27 in 1997Q4 to 12.40% in 2007Q2. From 2008 onward, the inflation rate increased remarkably from 4.8 % to 6.12% in 2013Q2. This could be

[1] The trimestriel unemployment rate in 1971 is unavailable.

explained by the revolution of 2011. Similarly, unemployment reached about 18.9% in 2011Q4.

We estimated the GARCH-DCC model during the period 1974Q1-2016Q1. We chose this period because we expected that the correlation dynamics between CPI inflation and unemployment rate during the stable in-sample periods would be significantly different from those during the out-of-sample periods.

Unlike the study of Russell and Chowdhury (2013), the present paper relied on an empirical methodology by applying Engle's (2002) DCC technique to describe the time-varying correlations between the CPI inflation and unemployment rates in Tunisia using a quarterly database 1974Q1-2016Q1. The choice of the study country was not random; it was rather a challenge. In fact, the used methodology is not very common among the developed countries, the authors thought it would be interesting to find the outcome of such an investigation in Tunisia, a developing country.

The remainder of this paper is structured as follows: Section 2 introduced the GARCH-DCC model. Section 3 detailed the used data in this study. Section 4 displayed and discussed the empirical results of the study and Section 5 drew the main conclusions and forwarded some recommendations.

2. Econometric model

Following the study of Hamilton (2008), we presented the CPI inflation and unemployment rate as a GARCH process. Further, we applied the DCC model introduced by Engle (2002). The dynamic conditional correlation generalized autoregressive conditional heteroskedasticity (GARCH-DCC) model is an econometric tool to describe the time varying correlations between two or more time series. The model's basis is to take the standardized residuals from GARCH volatility estimations to estimate the correlation.

The basis for the DCC model is Engle's original autoregressive conditional heteroskedasticity (ARCH) model (Engle, 1982), Bollerslev's generalized autoregressive conditional heteroskedasticity (GARCH) model (Bollerslev, 1986) and Bollerslev's constant conditional correlation (CCC) model (Bollerslev, 1990). Thus, the DCC model can be written as follows:

Let $y_t = [y_{1t}, y_{2t}]$ be a 2×1 vector that contains the time series of the CPI inflation and unemployment rate. The vector autoregression (VAR) can be presented by:

$$A(L)y_t = \varepsilon_t \quad (1)$$

Where $A(L)$ is a matrix in the lag operator L and $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}]$ is a vector of innovations.

The VAR (k) model (equation 1) indicates that the current movement of variables y_t can be explained by their own past movements $(y_{t-1}, y_{t-2}, \dots)$.

The innovation process is ruled by the following GARCH (p,q) process:

$$h_{i,j} = \omega_i + \sum_{i=1}^{p} \alpha_i \varepsilon_{i,t-1}^2 + $$
$$+ \sum_{i=1}^{q} \beta_i h_{i,t-1} \; (i = 1,2) \quad (2)$$

where E_{t-1} is the conditional information operator based on the information at time t-1.

The two parameters α and β are the GARCH-DCC model coefficients which govern the adjustment "speed" to new information (effect of structural breaks). In addition, this model (equation 2) is mean reverting as long as $\alpha + \beta < 1$.

The DCC was applied to illustrate the dynamic relationship between the CPI inflation and the unemployment rate. Specifically, relying on the volatilities to equation 2, we calculated the conditional correlations from the conditional covariance matrix as:

$$H_t = E[\varepsilon_t \varepsilon_t'] = D_t R_t D_t \quad (3)$$

where the diagonal matrix D_t represents the conditional volatilities from equation 2. R_t is the dynamic correlation matrix.

According to Engle (2002), the dynamic matrix process can be written as:

$$Q_t = (\bar{Q} - A'\bar{Q}A - B'\bar{Q}B) + + A'z_{t-1}z'_{t-1}A + B'Q_{t-1}B \quad (4)$$

where $\bar{Q}$ represents the unconditional correlation matrices of z_t.

Then, the conditional correlation matrix R_t can be presented as follows:

$$R_t = Q_t^{*-1}Q_tQ_t^{*-1} \quad (5)$$

where the diagonal matrix, which contains the square roots of the diagonal elements of Q_t, is written as $Q_t^* = \sqrt{q_{ii,t}}$.

Therefore, the bivariate DCC can be specified as:

$$q_{ij,t} = (\bar{q} - a^2\bar{q} - b^2\bar{q}) + + a^2z_{ij,t-1} + b^2q_{ij,t-1} \quad (6)$$

where $z_{i,t-1}$ is the time varying vector of the standardized residuals $\frac{\varepsilon_{i,t-1}}{h_{i,t-1}}$ and $a^2 + b^2 < 1$ is the restriction condition.

3. Empirical applications

Our purpose in this paper was to develop the dynamic relationship between the CPI inflation and the unemployment rates. This link remains one of the most commonly cited stylized facts in modern macroeconomics and remains of fundamental importance in monetary policy transmission.

3.1. Data Description and Sources

We used the quarterly consumer price index inflation noted by (π^1), with the 2010 serving as the base year, and the unemployment rate noted by (u) from the first quarter of 1974 up to the first quarter of 2016 in Tunisia. In fact, the CPI was collected from National Institute of Statistics (NIS) and for the total unemployment rate; we used the statistics provided by the National Institute of Competitiveness and Quantitative Studies (NICQS).

The descriptive statistics are reported in Table 1.

Table 1. *CPI inflation and Unemployment Rate Descriptive Statistics*

Statistics	Mean	Median	Max	Min	Standard Deviation	Skewness	Kurtosis	Jarque-Bera	Q(12)	Observations
CPI Inflation (π)	5.71	5.19	16.00	0.26	2.89	0.94	4.02	32.57***	806.1***	169
Unemployment rate (u)	13.89	14.19	18.88	11.40	1.38	0.77	4.64	35.97***	791.8***	169

Source: *Authors' calculation from the data source.*
Notes: **** represents significance at the 1% level. Q(12) is the Ljung–Box Q statistics for the null hypothesis that there is no autocorrelation up to order 12 for standardized residuals.*

Relying on the results in Table 1, the average CPI inflation of Tunisia is around 5.71% from the first quarter of 1974 to the first quarter of 2016. However, during the same period, Tunisia had maximum and minimum CPI inflation rates of 16.00% and 0.26%, respectively. Tunisia's average unemployment rate is approximately 13.85%. The unemployment rates range between a maximum of 18.88% and a minimum of 11.40%. The results of the Jarque–Bera test indicate that the null hypothesis of normality is rejected for all the cases, which indirectly supports the existence of ARCH effects. The results of the Ljung–Box Q statistics demonstrate that the null hypothesis of no autocorrelation up to order 12 is rejected at the 1% level for all the cases. In addition, the CPI inflation is positively skewed with a sample mean of 5.71% during this period, reflecting sharp CPI inflation increases during some crisis periods. The CPI inflation and the Unemployment rate data plots in the period are presented in Figure 1, where the left vertical axis is the CPI inflation scale ranging from zero to 18% and the right vertical axis shows the Unemployment rate scale. From Figure 1 we can see that the relationship between the CPI inflation and Unemployment rate appears to be nonlinear due to different economic circumstances. The CPI inflation and Unemployment rate features indicate that an appropriate model for the Phillips curve may be nonlinear.

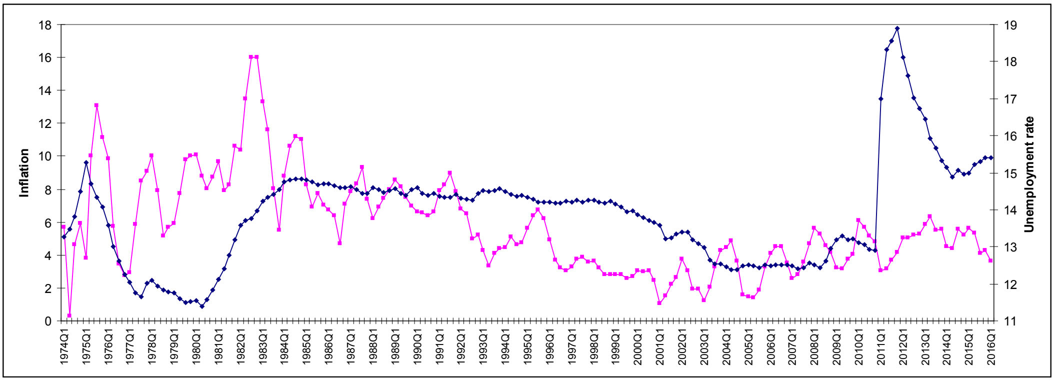

Fig 1: *Time series plots of the CPI inflation (pink) and the unemployment rate (blue)*

3.2. Unit root test

Before estimating our model using GARCH-DCC (dynamic conditional correlation) model, it is useful to carry out stationarity tests of the retained variables. In fact, we referred to Perron test (1997) to investigate the stationary status of each variable following its evolution over time (see Figure 1). The aspects of the series show a break in the slope and a break in the intercept. Then, the estimating equation that allows both effects (one time change in both the level and the slope of the series) is written as follows:

$$y_t = \alpha_0 + \varphi y_{t-1} + \delta_1 DU_t + \delta_2 DT_t + + d(DT_B)_t + \beta t + \sum_{i=1}^{p} \theta_i \Delta y_{t-1} + \varepsilon_t \qquad (7)$$

Where the intercept dummy DU_t represents a change in the level with $DU_t = 0 \; if \; t \leq T_B$ and $DU_t = 1 \; if \; t > T_B$. The slope dummy DT_t represents a change in the slope with $DT_t = 0 \; if \; t \leq T_B$ and $DT_t = t \; if \; t > T_B$. The crash dummy $DT_B = 1 \; if \; t = T_B + 1$ and zero otherwise and T_B is break date. The Perron (1997) test results of the unemployment rate and CPI inflation are shown in table 2.

Table 2. *Perron (1997) test results of the unemployment rate and CPI inflation*

Panel A: Unemployment rate (u)			Panel B: CPI inflation (π)		
Variables	*Coefficient*	*t-statistics*	*Variables*	*Coefficient*	*t-statistics*
Constant	1.767	4.85	Constant	2.063	3.981
DU	10.261	4.85	DU	-2.117	-3.187
DTB	-1.166	-3.04	DTB	0.322	0.405
@trend	6.1e-5	0.09	@trend	-0.010	-2.253
DT	-0.062	-4.67	DT	0.016	2.596

u{1}	0.868	32.99	π{1}	0.801	15.949
TB	t-statistics	Critical Value	TB	t-statistics	Critical Value
2010:03	-5.01	-5.08	1995:03	-3.95	-5.08
Panel C: Unemployment rate (Δu)			**Panel D: CPI inflation (Δπ)**		
Constant	0.121	1.55	Constant	-1.152	-1.23
DU	18.158	6.34	DU	-0.094	-0.10
DTB	-1.580	-3.66	DTB	3.546	3.37
@trend	-0.001	-1.62	@trend	0.079	2.28
DT	-0.111	-6.28	DT	-0.069	-2.02
Δu{1}	0.392	4.25	Δπ{1}	-0.182	-1.24
TB	t-statistics	Critical Value	TB	t-statistics	Critical Value
2010:03	-6.60	-5.08	1983:01	-8.07	-5.08

Note : Author's *estimation using*

Based on Perron test (1997), the results in table 2 show that the CPI inflation and unemployment rate contain a unit root. Also, this test indicates that the unemployment rate is stationary at first difference with a significant structural break occurring in the third quarter of 2010 and the CPI inflation is also stationary at first difference with a significant structural break occurring in the first quarter of 1983.

4. Empirical results

The results in Table 3 show that the estimates in the variance equation are statistically significant at the 1% level, and they satisfy the restrictions of $\omega>0$, $\alpha>0$, $\beta>0$, $\alpha+\beta<1$ which confirms that the GARCH-type models are appropriate. The coefficients in the DCC model are also estimated to be statistically significant at the 1% level, and these also satisfy the restrictions of $a^2+b^2<1$. When $\alpha+\beta<1$, indicating that the GARCH-DCC technique is mean back. In the GARCH-DCC technique, the estimated parameter α is smaller than β, which means that CPI inflation and unemployment are inversely correlated. The Ljung–Box Q statistics also suggest that the empirical results of the models have been adequately estimated.

Table 3. *Bivariate GARCH-DCC estimation*

	CPI Inflation variation ($\Delta\pi$)	Unemployment variation (Δu)
Mean equation		
Δu_{t-1}	0.136 (0.054)**	1.296 (0.084)***
Δu_{t-2}	-0.156 (0.057)**	-0.318 (0.116)***
Δu_{t-3}	-	-0.039 (0.100)
Δu_{t-4}	-	-0.034 (0.005)***
$\Delta \pi_{t-1}$	0.699 (0.076)***	-0.003 (0.001)***
$\Delta \pi_{t-2}$	-0.239 (0.087)***	0.008 (0.005)
$\Delta \pi_{t-3}$	-0.033 (0.079)	-0.010 (0.005)*
$\Delta \pi_{t-4}$	-0.409 (0.074)***	0.008 (0.004)**
d1983q1	-0.345 (0.036)***	-
d2010q3	-	0.023 (0.004)***
constant	0.273 (0.347)	0.0005 (0.007)
Variance equation		
ω	0.110 (0.033)***	0.003 (0.001)***
α	0.798 (0.103)***	0.289 (0.120)**
β	0.228 (0.023)***	0.628 (0.110)***
DCC		
a^2	0.0110 (0.0036)***	
b^2	0.9431 (0.0889)***	
Diagnostic		
Q(1)	0.193 [0.661]	3.671 [0.055]
Q(12)	8.177 [0.771]	8.245 [0.235]
Q(24)	9.438 [0.994]	9.035 [0.329]
$Q^2(1)$	0.011 [0.916]	3.540 [0.060]
$Q^2(12)$	0.231 [1.000]	9.229 [0.987]
$Q^2(24)$	0.253 [1.000]	9.518 [0.999]

Notes: *The numbers in parentheses are standard errors. The numbers in square brackets are p-values. Q(1), Q(12) and Q(24) ($Q^2(1)$, $Q^2(12)$ and $Q^2(24)$) are the Ljung–Box Q statistics for the null hypothesis that there is no autocorrelation up to orders 1, 12 and 24 for the standardized residuals (standardized squared residuals), respectively. *, ** and *** represent the significance at the 10%, 5% and 1% levels, respectively.*
Author's estimation using STATA 13.0

The bivariate GARCH-DCC model estimation results reveal that the sign of the correlation between CPI inflation and unemployment rate changed from negative to positive spanning from 1974Q1 to 2016Q1. This result indicates that the relationship between CPI inflation and unemployment rate is not invariant of economic policies, thus providing evidence that supports Lucas critique. These results also reveal that the shape of the Phillips curve is not independent of economic environment.

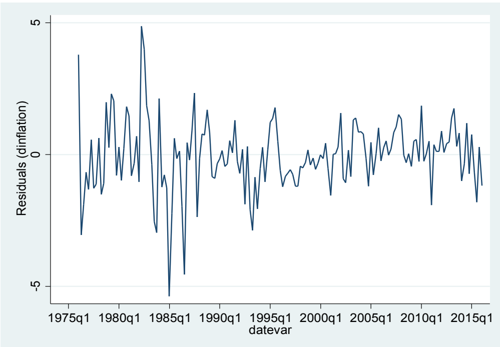

Fig 2. *Time series plots of the CPI inflation residual*

Figures 2 and 3 enable us to observe the significance of the residuals of the bivariate GARCH-DCC model including the dummy variables representing the two chocks. It is clear that the two dummy variables are significant in the model that captured the volatility problem. In fact, the CPI inflation choc was relative to Tunisian economic crisis during the 1980s and the employment choc was caused by the 2011 Tunisian revolution. The high volatility of the CPI inflation and unemployment rate is clear especially around the shocks' periods. This proves the importance of the GARCH-DCC model to detect the volatility problem of the two retained series. This may help the authorities to pay attention to decision grip.

As expected, during the simultaneous oil price shaking the 1970s and the recession periods (1982-1986), the volatility between the CPI inflation and unemployment rate increased. During the subsequent two recession periods, the volatility fell but increased again after the revolution period in 2011.

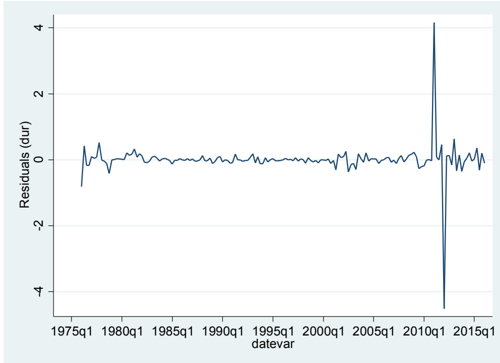

Fig 3. *Time series plots of the unemployment rate residual*

Conclusion

Applying the GARCH-DCC model, we re-examined the relationship between CPI inflation and unemployment rate in Tunisia from 1974Q1 to 2016Q1 in this study. Compared to previous studies, the empirical results revealed a negative correlation during the periods of contraction and recession but a positive correlation during periods of relatively rapid economic growth. The results in this study proved that the Phillips curve in Tunisia is rather unstable in the long run.

Our results suggest two main implications for policymakers. First, monetary policy may have more important and long-lasting effects on unemployment during a recession period compared to a boom period. Second, since keeping the unemployment rate at a reasonable level is the priority of the monetary authorities, the trade-off between CPI inflation and unemployment rate must be evaluated before implementing any policy, especially when the economy is strong. The framework makes it possible to identify the channels of these effects: CPI inflation reduces growth by reducing investment and productivity growth;

budget deficits also reduce both capital accumulation and productivity growth. In addition, a low CPI inflation and small deficits are not necessary for a high growth even over long periods; a high CPI inflation is not consistent with a sustained growth.

Pinpointing the factors that cause the change in the correlation between CPI inflation and unemployment rate would be an interesting trend for future research.

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