Articles

Exchange Values of Gold, Land, Physical Capital, and Human Capital in a Neoclassical Growth Model

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Summary:

Households of contemporary economies are characterized by many assets such as gold, silver, diamond, land, housing, physical wealth, in their portfolios. Yet modern dynamic economic theory has only a few mathematical models with physical wealth, land and gold based on microeconomic foundation. This study deals with portfolio choice equilibrium with land, gold and physical capital, and human capital within a general equilibrium framework. The growth determinants are endogenous physical and human wealth accumulation. The model studies dynamic interactions between land value, gold value, economic structural change, wealth accumulation and human capital growth by integrating the neoclassical growth theory, the Ricardian theory, and the Uzawa-Lucas model with Zhang's utility function. The human capital accumulation is due to Arrow's learning by doing, Uzawa's learning through education, and Zhang's learning through consuming (leisure creativity). We simulate the motion of the economic system and demonstrate the existence of a unique stable equilibrium point. Comparative dynamic analysis is conducted with regard to exogenous changes in the propensity to consume gold, the propensity to receive education, the propensity to consume housing, the propensity to save, and the population.

Key words: gold value; land value; human capital accumulation; education; wealth accumulation.

JEL Classification: O41, E25, E21

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1. Introduction

In association with modern economic growth and structural changes, the complexity of portfolio is increased (e.g., Uhler and Gragg, 1971; Agell and Edin, 1990; Cobb-Clark and Hilderbrand, 2009; Gaudecker, 2015). Households of contemporary economies are characterized by many assets such as housing, land, stocks, precious metals, gold, cashes in different currencies in their portfolios. It is obvious that genuine dynamic theories are needed in order to properly analyze portfolio equilibrium dynamics in association with economic growth and structural changes. As summarized in the Overview by Guiso

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et al. (2002), "Until recently, researchers in economics and finance paid relatively little attention to household portfolios. Reasons included the tendency of most households to hold simple portfolios, the inability of the dominant asset pricing models to account for household portfolio incompleteness, and the lack of detailed databases on household portfolios in many countries until the late 1980s or 1990s. Now, however, the analysis of household portfolios is emerging as a field of vigorous study." Modern economic theory has only a few mathematical models with household portfolios based on microeconomic foundation. This study proposes a mathematical model to deal with growth with portfolio choice equilibrium with land, gold and physical capital, and human capital within a comprehensive framework. The model provides some theoretical insights into the complexity of dynamic interactions between the household portfolios and economic growth.

This paper aims to connect land and gold value determination with accumulation of wealth and human capital. The economic aspects are strongly influenced by the neoclassical growth theory. The theory is mainly concerned with endogenous physical capital or wealth accumulation (Solow, 1956; Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). We follow the traditional neoclassical growth theory in modeling economic production and wealth accumulation. In addition to capital accumulation, human capital accumulation is another key engine mechanism of economic growth (Mincer, 1974; Easterlin, 1981; Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; CastellÓ-Climent and Hidalgo-Cabrillana, 2012; and Barro and Lee, 2013). There is a large number of theoretical works on endogenous knowledge and economic growth (Romer, 1986; Lucas, 1988; Grossman and Helpman, 1991; and Aghion and Howitt, 1998; Chen and Chevalier, 2008). The early formal dynamic growth model with education was proposed by Uzawa (1965). Lucas' paper in 1988 aroused a great interest in modelling education and economic growth. The Uzawa-Lucas model and many of their extensions and generalizations consider skills and human capital accumulation due to formal education. Nevertheless, it is common sense to argue that formal education is one way of accumulating human capital. Except formal education much of the human capital may be accumulated within family and through other social and economic activities. Ignoring non-school factors in determining human capital may over-emphasize the role of formal education in economic development. This study takes account of three ways of learning: learning through education as in the Uzawa-this study, learning through doing as in Arrow (1962), and learning through creative leisure as in Zhang (2007) in modeling human capital accumulation.

Most studies in the neoclassical growth theory omit determinants and dynamics of land value. In *On the Principles of Political Economy and Taxation* published in 1817, Ricardo examined the links between wages, interest rate, and rent by distinguishing between the three production factors, labor, capital, and land. Since the publication of the *Principles*, economists have made a lot of efforts to extend or generalize the Ricardian system (see Barkai, 1959, 1966; Pasinetti, 1960, 1974; Cochrane, 1970; Brems, 1970; Caravale and Tosato, 1980; Casarosa, 1985; Negish, 1989; Morishima, 1989). Nevertheless, what Ricardo described a long time ago is still valid for the current state of the literature: "To determine the laws which regulate this distribution, is the principal problem in Political Economy: much as the science has been improved by the writings of Turgot, Stuart, Smith, Say, Sismondi, and others, they afford very little satisfactory information respecting the natural course of rent, profit, and wages." (Ricardo, 1821: preface). As recently reviewed by Liu *et al*. (2011: 1), "Although it is widely accepted that house prices could have an important influence on macroeconomic fluctuations, quantitative studies in a general equilibrium framework have been scant." In the contemporary literature on land and economic growth only a few studies are concerned with determining land value and economic growth with microeconomic foundation. This study is concerned not only with land value and economic growth, but also with gold value, economic growth and economic structural change. Dynamics of gold prices are well mentioned but not properly theoretically examined (Barro, 1979; Bordo and Ellson, 1985; Dowd and Sampson, 1993; Chappell and Dowd, 1997). Through building a general equilibrium model, we study the dynamic interdependence between land value, land rent, gold value, wage rate, rate of interest, physical wealth value, and human capital value with endogenous wealth and human capital.

This study is a synthesis of Zhang's recent two models (Zhang, 2015, 2016). Zhang (2015) builds a model with endogenous education, while Zhang (2016) constructs a growth model of portfolio equilibrium choice among land, gold and physical wealth. As far as portfolio equilibrium is concerned, this paper follows Zhang's 2016 paper. This study makes human capital endogenous by introducing education sector on the basis of Zhang's 2015 paper. This paper is organized as follows. Section 2 develops the growth model with endogenous physical and human capital accumulation. Section 3 examines dynamic properties of the model and simulates the model. Section 4 carries out a comparative dynamic analysis with regard to changes in the propensity to receive education, the propensity to consume housing, the propensity to consume industrial goods, the propensity to consume agricultural goods, the propensity to save, the total factor productivity of the industrial sector, and the population. Section 5 concludes the study. The appendix proves the results in section 3.

2. The model

This study deals with a dynamic economy in the industrial, agricultural and education sectors. The industrial sector produces goods, the agricultural sector produces agricultural goods, and the education sector conducts education. The industrial production is the same as that in Solow's one-sector neoclassical growth model. The industrial product is used for investment and consumption. The agricultural product is for consumption. The education sector supplies education and output is human capital accumulation. The education sector is based on the Uzawa-Lucas two-sector models (Uzawa, 1965; Lucas, 1988). Most aspects of the production sectors are similar to those in the neoclassical growth models (Barro and Sala-i-Martin, 1995). Both physical wealth (capital) and human capital are endogenous. The growth of physical capital is due to saving and the growth of human capital is through learning by education, learning by producing, and learning through creative leisure. All the markets are perfectly competitive. The available input factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. We select industrial goods to serve as numeraire. The population $\bar{N}$ is homogenous and constant. The qualified labor supply is dependent on the work time and human capital. The time distribution between work and study and the household's human capital are endogenous. Households own assets of the economy and distribute their incomes to receive education, to consume and to save. Firms use labor and physical capital inputs to supply goods and services. Let $H(t)$ stand for the level of human capital in time t. The total amounts of land and gold are fixed and privately owned by households. Land and gold can be sold and bought in free markets without any friction and transaction costs. Land and gold do not depreciate. Households can own land, gold and physical wealth. The total land L is homogenous and constant. The land is distributed between housing and agricultural production.

We use subscript index, i, a, and e, to stand for industry, agriculture, and education, respectively. We use $N_j(t)$ and $K_j(t)$ to stand for the labor force and capital stocks employed by sector j, $j = i$, a, and e. We use $T(t)$ and $T_e(t)$ to represent, respectively, the work time and study time of a representative worker. Markets are competitive; thus labor and capital earn their marginal products. The rate of interest $r(t)$ and wage rate $w(t)$ are determined by markets. The variable $N(t)$ is used to stand for the total qualified labor force. A worker's labor force is $T(t)H^m(t)$, where m is a parameter measuring the utilization efficiency of human capital. The total labor input is the sum of all the households' labor inputs, i.e.

$$N(t) = T(t)H^m(t)\overline{N}. \tag{1}$$

We use $F_j(t)$ to represent the production function of sector j, $j = i, a, e$.

The industrial sector

We assume that production is to combine the qualified labor force, $N_i(t)$, and physical capital, $K_i(t)$. The production function $F_i(t)$ is

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \ \ \alpha_i, \beta_i > 0, \ \ \alpha_i + \beta_i = 1, \tag{2}$$

where A_i, α_i, and β_i are positive parameters. The profit of the sector is

$$\pi_i(t) = F_i(t) - (r(t) + \delta_k)K_i(t) - w(t)N_i(t),$$

where δ_k is the fixed depreciation rate of physical capital. The first-order conditions for the capital goods sector are

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \ \ w(t) = \frac{\beta_i F_i(t)}{N_i(t)}. \tag{3}$$

The agricultural sector

The agricultural sector supplies goods $F_a(t)$ by combining capital $K_a(t)$, labor force $N_a(t)$, and land $L_a(t)$ in the following way

$$F_a(t) = A_a K_a^{\alpha_a}(t) N_a^{\beta_a}(t) L_a^{\varsigma}(t), \ \ A_a, \alpha_a, \beta_a, \varsigma > 0, \ \ \alpha_a + \beta_a + \varsigma = 1, \tag{4}$$

where $L_a(t)$ is the land employed by the agricultural sector, and A_a, α_a, β_a, and ς are parameters. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_a p_a(t) F_a(t)}{K_a(t)}, \ \ w(t) = \frac{\beta_a p_a(t) F_a(t)}{N_a(t)}, \ \ R_L(t) = \frac{\varsigma p_a(t) F_a(t)}{L_a(t)}, \tag{5}$$

where $p_a(t)$ stands for the price of agricultural goods and $R_L(t)$ stands for the land rent.

The education sector

The education sector is perfectly competitive and is described in Zhang (2015). Let $p_e(t)$ represent the student's education fee per unit of time. The education sector pays teachers and capital owners the market rates. The total education service is measured by the total education time received by the population. The production function of the education sector is

$$F_e(t) = A_e K_e^{\alpha_e}(t) N_e^{\beta_e}(t), \ \alpha_e, \beta_e > 0, \ \alpha_e + \beta_e = 1, \tag{6}$$

where A_e, α_e and β_e are positive parameters. We use $p_e(t)$ to stand for the price of per unit time of education. The marginal conditions for the education sector are

$$r(t) + \delta_k = \frac{\alpha_e p_e(t) F_e(t)}{K_e(t)}, \ w(t) = \frac{\beta_e p_e(t) F_e(t)}{N_e(t)}. \tag{7}$$

The demand for labor force from the education sector increases in the price and level of human capital and decreases in the wage rate.

Choice between physical wealth, gold, and land

Let $p_L(t)$ and $p_G(t)$ stand for, respectively, the prices of land and gold. For convenience of analysis, it is assumed that gold can be "rented" through markets for decoration use. The rent of gold is denoted by $R_G(t)$. The gold owned by the representative household is assumed to be fully used either by the household for decoration or rented out to other households. Consider now a household with one unity of money. He can either invest in capital good thereby earning a profit equal to the net own-rate of return $r(t)$ or invest in land (gold) thereby earning a profit equal to the net own-rate of return $R_L(t)/p_L(t)$ ($R_G(t)/p_G(t)$). As we assume capital, gold and land markets to be at competitive equilibrium at any point in time, three options yield equal returns, i.e.

$$\frac{R_G(t)}{p_G(t)} = \frac{R_L(t)}{p_L(t)} = r(t). \tag{8}$$

These equations enable us to determine the portfolio equilibrium choice between gold, land and (physical) wealth. It is easy to see that equations (8) are established under many strict conditions. For instance, we omit any transaction costs and any time delay for buying and selling. Equation (8) also implies perfect information.

The current income and disposable income

As in many studies in urban economics (e.g., Zhang, 2002), we use lot size to stand for housing. As argued, for instance, by Davis and Heathcote (2007), most of the fluctuations in house prices are driven by land price rather than by the cost of structures. This implies that it is acceptable to use lot size to stand for housing when dealing with dynamics of housing value. Consumers decide consumption levels of education, industrial good, agricultural good, gold and lot size, as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang (1993, 2005). Let $\bar{k}(t)$ stand for the representative household's physical wealth, $\bar{g}(t)$ for the amount of gold owned by the household, and $\bar{l}(t)$ for the lot size. The total value of wealth owned by the household $a(t)$ is the sum of the three assets' values

$$a(t) = \bar{k}(t) + p_L(t)\bar{l}(t) + p_G(t)\bar{g}(t). \tag{9}$$

We now define the representative household's current income. It is the sum of the interest payment of physical wealth $r(t)\bar{k}(t)$, the wage payment $H^m(t)T(t)w(t)$, the gold interest income $R_G(t)\bar{g}(t)$ and the land revenue $R_L(t)\bar{l}(t)$ as follows

$$y(t) = r(t)\bar{k}(t) + H^m(t)T(t)w(t) + R_L(t)\bar{l}(t) + R_G(t)\bar{g}(t). \tag{10}$$

The household's disposable income is the sum of the current income and value of the household's assets, i.e.

$$\hat{y}(t) = y(t) + a(t). \tag{11}$$

The budget, utility function, and optimal decision

The disposable income is used for saving and consumption. At each point in time, the representative household would distribute the total available budget between education $T_e(t)$, consumption of industrial goods $c(t)$, consumption of agricultural goods $c_a(t)$, use of gold for decoration $\hat{g}(t)$, lot size $l_h(t)$, and saving $s(t)$. The budget constraint is given by

$$p_e(t)T_e(t) + c(t) + s(t) + p_a(t)c_a(t) + R_G(t)\hat{g}(t) + R_L(t)l_h(t) = \hat{y}(t). \tag{12}$$

The available time is distributed between education and work. The time constraint for everyone is

$$T(t) + T_e(t) = T_0, \tag{13}$$

where T_0 is the total available time. Insert (13) in (10) and then insert the resulted equation to (12) to get

$$\bar{p}_e(t)T_e(t) + c(t) + s(t) + p_a(t)c_a(t) + R_G(t)\hat{g}(t) + R_L(t)l_h(t) = \bar{y}(t), \tag{14}$$

where

$$\bar{p}_e(t) \equiv p_e(t) + H^m(t)w(t), \quad \bar{y}(t) \equiv (1 + r(t))a(t) + H^m(t)T_0 w(t).$$

The representative household has six choice variables, $s(t)$, $T_e(t)$, $c(t)$, $c_a(t)$, $l_h(t)$, and $\hat{g}(t)$. It can be seen that $\bar{p}_e(t)$ represents the opportunity cost of education.

As in Zhang (2015), the consumer's utility function is specified as follows

$$U(t) = T_e^{\kappa_0}(t)c^{\xi_0}(t)c_a^{\mu_0}(t)l_h^{\eta_0}(t)\hat{g}^{\gamma_0}(t)\, s^{\lambda_0}(t), \quad \kappa_0, \xi_0, \mu_0, \eta_0, \gamma_0, \lambda_0 > 0,$$

in which κ_0, ξ_0, μ_0, η_0, γ_0, and λ_0 are the household's elasticity of utility with regard to education, industrial good, agricultural good, housing, gold decoration, and saving. We call κ_0, ξ_0, μ_0, η_0, γ_0, and λ_0 propensities to consume industrial goods, agricultural goods, housing, and to hold wealth, respectively. Education has been modeled in different ways (Becker, 1981; Cox, 1987; Behrman *et al*. 1982; Fernandez and Rogerson, 1998; Banerjee, 2004; Florida, et al. 2008; Galindev, 2011). According to Lazear (1977: 570), "education is simply a normal consumption good and that, like all other normal goods, an increase in wealth will produce an increase in the amount of schooling purchased. Increased incomes are associated with higher schooling attainment as the simple result of an income effect." It is argued that a rise in education tends to result in higher wages (e.g., Heckman, 1976;

Lazear, 1977; Malchow-Møller, *et al.* 2011). Education also brings about direct pleasure, more knowledgeable, higher social status and so on. The introduction of education time into the utility function implies that the household decides the time spent on education due to the welfare that education may bring about, not only due to the increased wage rate associated with higher education.

Maximizing $U(t)$ subject to (14) yields

$$(t)T_e(t) = \kappa\,\bar{y}(t),\;\; c(t) = \xi\,\bar{y}(t),\;\; p_a(t)c_a(t) = \mu\,\bar{y}(t),\;\; R_L(t)l_h(t) = \eta\,\bar{y}(t),\\ (t)\hat{g}(t) = \gamma\,\bar{y}(t),\;\; s(t) = \lambda\,\bar{y}(t), \tag{15}$$

where

$$\kappa \equiv \rho\kappa_0,\;\; \xi \equiv \rho\xi_0,\;\; \mu \equiv \rho\mu_0,\;\; \eta \equiv \rho\eta_0,\;\; \gamma \equiv \rho\gamma_0,\;\; \lambda \equiv \rho\lambda_0,\;\; \rho \equiv \frac{1}{\kappa_0 + \xi_0 + \mu_0 + \eta_0 + \gamma_0 + \lambda_0}.$$

Wealth accumulation

According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{a}(t) = s(t) - a(t). \tag{16}$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

Dynamics of human capital

In the traditional mathematical growth theory, human capital is accumulated either through learning by doing or through education (as well as training). The growth theory with learning by doing is initiated by Arrow (1962). Uzawa (1965) introduced education into the neoclassical growth theory. Zhang (2007) introduced impact of consumption on human capital accumulation (via the so-called creative leisure) into growth theory. Zhang (2007) integrated the three sources of human capital into a single analytical framework. According to Zhang (2007), the human capital dynamics is given by

$$\dot{H}(t) = \frac{\upsilon_e F_e^{a_e}(t)\left(H^m(t)T_e(t)\bar{N}\right)^{b_e}}{H^{\pi_e}(t)\bar{N}} + \frac{\upsilon_i F_i^{a_i}(t)}{H^{\pi_i}(t)\bar{N}} + \frac{\upsilon_h C^{a_h}(t)}{H^{\pi_h}(t)\bar{N}} - \delta_h H(t), \tag{17}$$

where $\delta_h\ (> 0)$ is the depreciation rate of human capital, $\upsilon_e,\ \upsilon_i,\ \upsilon_h,\ a_e,\ b_e,\ a_i$, and a_h are non-negative parameters. The signs of the parameters π_{je}, π_{ji}, and π_{jh} are not specified as they may be either negative or positive. The above equation is a synthesis and generalization of Arrow's, Uzawa's, and Zhang's ideas about human capital accumulation. The term $\upsilon_e F_e^{a_e}\left(H^m T_e \bar{N}\right)^{b_e} / H^{\pi_e}\bar{N}$ implies that the contribution to human capital improvement through education (e.g., Uzawa, 1965; Lucas, 1988; Barro and Sala-i-Martin, 1995; and Solow, 2000). Human capital increases in the level of education service F_e, and in the (qualified) total study time, $H^m T_e \bar{N}$. The population $\bar{N}$ in the denominator measures the contribution in terms of per capita. The term H^{π_e} indicates that as the level of human capital of the population increases, it may be more difficult (in the case of π_e being large) or easier (in the case of π_e being small) to accumulate more human capital via formal education. The term $\upsilon_i F_i^{a_i} / H^{\pi_i}\bar{N}$ implies the learning by producing effects in human capital accumulation. The term $\upsilon_h C^{a_h} / H^{\pi_h}\bar{N}$ takes account of learning by consuming.

Demand of and supply for education

The demand for education is $T_e(t)\overline{N}$ and the supply of education service is $F_e(t)$. The balance of demand for and supply of education at any point in time is

$$T_e(t)\overline{N} = F_e(t). \tag{18}$$

Balances of demand and supply for agricultural goods

The balance condition for demand for and supply of the agricultural good at any point in time is

$$C_a(t) = c_a(t)\overline{N} = F_a(t). \tag{19}$$

The land and the gold owned by households

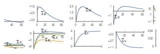
The assets owned by the population are equal to the available amounts of the assets

$$\bar{l}(t)\overline{N} = L. \tag{20}$$

$$\bar{g}(t)\overline{N} = G. \tag{21}$$

Gold being fully used for decoration

The amount of gold used for decoration by the population is equal to the total gold



(22)

Full employment of capital

We use $K(t)$ to stand for the total capital stock. The assumption that the capital stock is fully employed implies

$$K_i(t) + K_a(t) + K_e(t) = K(t). \tag{23}$$

The value of physical wealth and capital

The value of physical capital is equal to the value of physical wealth

$$\bar{k}(t)\overline{N} = K(t). \tag{24}$$

Full employment of labor force

We assume that labor force is fully employed

$$N_i(t) + N_a(t) + N_e(t) = N(t). \tag{25}$$

The land market clearing condition

The land is fully used

$$l_h(t)\overline{N} + L_a(t) = L. \tag{26}$$

We completed the model. The model is structurally general as some well-known models in economic theory can be considered as special cases of this model. For instance, if we fix wealth and human capital, then the model is a Walrasian general equilibrium model with a single type of household and heterogeneous production sectors. Our model is structurally similar to the neoclassical growth model by Solow (1956). The model is built on the basis of Arrow's learning by doing and the Uzawa-Lucas model. The rest of this paper examines dynamic properties of the model.

3. The dynamics and the motion by simulation

The economic system contains many variables which are nonlinearly interdependent. As it is difficult to solve the model analytically, we solve the model numerically. In the appendix, we show that the dynamics of the national economy can be expressed as two differential equations. First, we introduce a variable $z(t)$ by

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}.$$

The following lemma is proved in the appendix.

Lemma

The following two differential equations with $z(t)$ and $H(t)$ as the variables describe the motion of the economic system

$$\dot{z}(t) = \Lambda(z(t), H(t)),$$
$$\dot{H}(t) = \Omega(z(t), H(t)), \tag{27}$$

where Λ and Ω are functions of $z(t)$ and $H(t)$ given in the appendix. The following procedure determines the motion of all the other variables as functions of $z(t)$ and $H(t)$: $r(t)$ and $w(t)$ by (A2) → $\bar{k}(t)$ by (A27) → L_a and l_h by (A9) → $p_e(t)$ by (A10) → $\overline{p}_e(t)$ by (A11) → $p_a(t)$ by (A5) → $\overline{y}(t)$ by (A13) → $R_L(t)$ and $R_G(t)$ by (A14) → $p_L(t)$ and $p_G(t)$ by (A8) → $K_i(t)$ and $K_e(t)$ by (A23) → $K_a(t)$ by (A20) → $N_i(t)$, $N_e(t)$, and $N_a(t)$ by (A1) → $N(t)$ by (A17) → $a(t)$ by (A27) → $F_i(t)$ by (2) → $F_a(t)$ by (4) → $F_e(t)$ by (6) → $T_e(t)$, $c_i(t)$, $c_a(t)$, and $s(t)$ by (15) → $T(t)$ by (13) → $C_a(t)$ by (19) → $C_i(t) = c_i(t)\overline{N}$ → $\bar{l}$ by (20) → $\overline{g}$ by (21).

The lemma states that if we have the initial values of the two variables $z(0)$ and $H(0)$, we can determine all the variables of the economic system. The lemma enables us to follow the motion of the system with a computer. As the expressions of the analytical results are tedious, we simulate the model. We specify the parameters as follows

$$N = 10,\ T_0 = 24,\ L = 100,\ m = 0.6,\ \alpha_i = 0.29,\ \alpha_e = 0.3,\ \alpha_a = 0.1,\ \beta_a = 0.2,\ A_i = 1,$$
$$A_a = 0.8,\ A_e = 1,\ \lambda_0 = 0.8,\ \xi_0 = 0.07,\ \mu_0 = 0.04,\ \eta_0 = 0.07,\ \kappa_0 = 0.01,\ \gamma_0 = 0.01,$$
$$v_e = 0.8,\ v_i = 0.1,\ v_h = 0.2,\ a_e = 0.3,\ b_e = 0.5,\ a_i = 0.4,\ a_h = 0.1,\ b_e = 0.5,\ \pi_e = 0.5,$$
$$\pi_i = 0.5,\ \pi_h = 0.3,\ G = 0.5,\ \delta_k = 0.05,\ \delta_h = 0.03. \tag{28}$$

The population is fixed at 10 and the land is 100. The propensity to save is much higher than the propensity to consume industrial goods, the propensity to consume lot size, the propensity to use gold, and the propensity to consume agricultural goods. The depreciation rates of physical capital and human capital are respectively 0.05 and 0.03. The three sources of human capital accumulation exhibit decreasing return to scales. The amount of gold is $G = 0.5$. This number is not significant in the sense that a change in the amount only affects the gold price and rent and has no effects on the other variables. As shown in the appendix, the following variables are invariant in time

$$l_h = 7.14,\ L_a = 28.57,\ \bar{l} = 10,\ \overline{g} = \hat{g} = 0.05.$$

We specify the following initial conditions

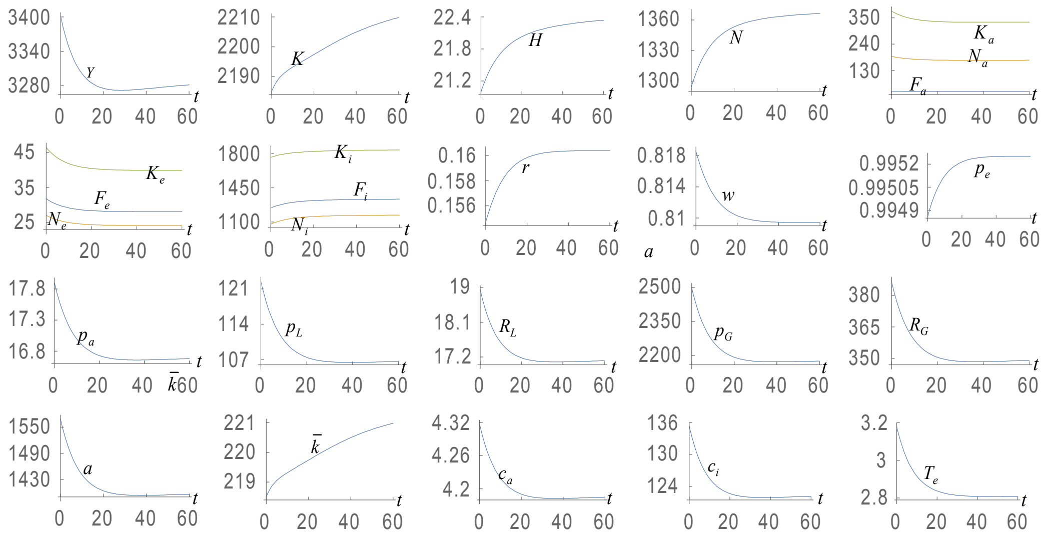
$$z(0) = 0.25,\ H(0) = 21.$$

We plot the motion of the variables in Figure 1. In Figure 1 the national gross product (GDP) is

$$Y(t) = F_i(t) + p_a(t)F_a(t) + p_e(t)F_e(t) + l_h \overline{N} R(t).$$

The GDP falls over time till it achieves the long-term equilibrium value. The national capital stock, human capital and total labor force are augmented. The wage rate falls and the rate of interest rises. The prices of land and gold and rents of land and gold are reduced. The price of agricultural goods falls and the price of education rises. The output and input factors of the industrial sector are enhanced. The output levels and input factors of the other two sectors are reduced. The household works more hours and studies less. The household's physical wealth rises and the total wealth falls. The household also consumes less agricultural goods and industrial goods.

Fig.1. The Motion of the Economic System



From Figure 1 we observe that all the variables become stationary in the long term. This implies the existence of some equilibrium point. We confirm the existence of the equilibrium point as follows

$$Y = 3291.7,\ K = 2216.1,\ H = 22.4,\ w = 0.81,\ p_L = 107.1,\ R_L = 17.2,\ p_G = 2184.6,$$
$$R_G = 350.3,\ r = 0.16,\ p_a = 16.7,\ p_e = 0.995,\ F_a = 41.9,\ F_i = 1336.9,$$
$$F_e = 28.1,\ K_a = 333.1,\ K_i = 1843.1,\ K_e = 39.9,\ N_a = 173.1,\ N_i = 1172.6,\ N_e = 24.2,$$
$$\bar{k} = 221.6,\ c_a = 4.2,\ c_i = 122.6,\ a = 1401.3,\ T_e = 2.81. \quad (29)$$

The eigenvalues at the equilibrium point are

$$-0.114,\ -0.029.$$

The equilibrium point is locally stable. The stability guarantees that we can effectively conduct a comparative dynamic analysis.

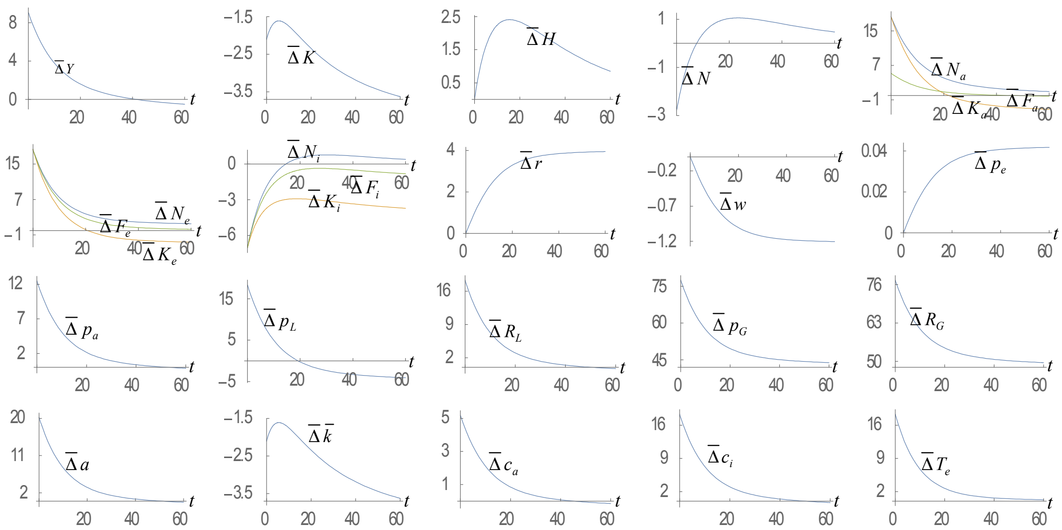
4 Comparative dynamic analysis

We now examine effects of changes in some parameters on the motion of the economic system. As the lemma gives a computational procedure to calibrate the motion of all the variables and the equilibrium point is locally stable, it is straightforward for us to plot the impact of any change in a parameter on transitory processes and long-term equilibrium. In the rest of this study we use $\overline{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in a parameter value.

A rise in the propensity to use gold

First we allow the propensity to use gold to be enhanced as follows: $\gamma_0 : 0.01 \Rightarrow 0.015$. We note that the lot size, agricultural land-use, and gold per capita are not affected, i.e., $\overline{\Delta} l_h = \overline{\Delta} L_a = \overline{\Delta} \overline{g} = 0$. The motion of the variables are plotted in Figure 2. The gold price and gold rent are enhanced. The total capital and physical wealth per household are reduced. The human capital is increased. The total labor force and GDP rise initially and fall in the long term. The labor inputs of all the three sectors are increased in the long term and the capital inputs of all the three sectors are decreased in the long term. The output levels of the agricultural and education sectors are slightly affected in the long term. The output level of the industrial sector is reduced. The rate of interest falls in tandem with rising in the wage rate. The price of agricultural goods rises initially and changes slightly in the long term. The education fee is slightly affected. The land price is increased initially and reduced in the long term. The land rent rises initially and changes slightly in the long term. The consumption levels of agricultural and industrial goods and education time rise initially and change slightly in the long term.

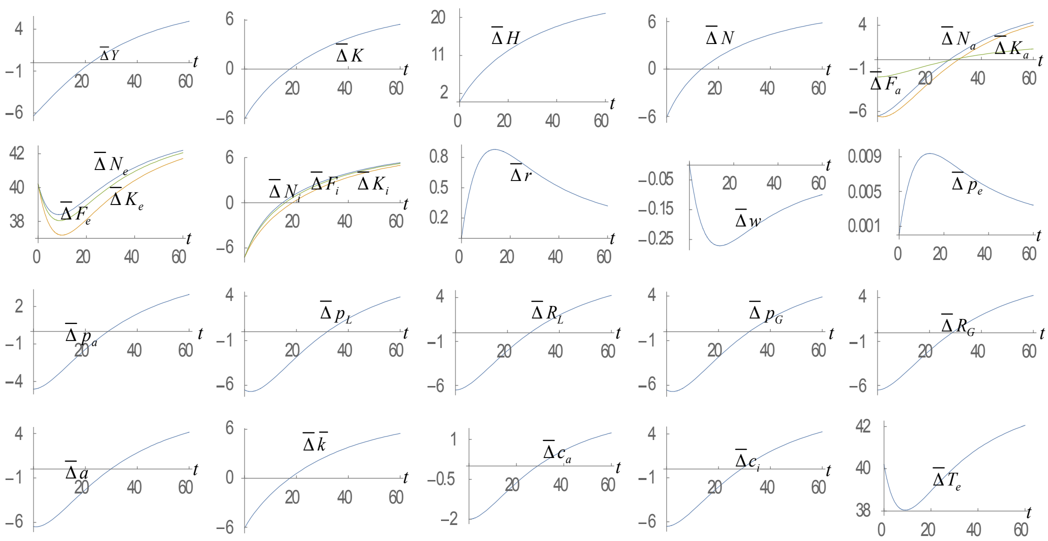
Fig.2. A Rise in the Propensity to Use Gold



The propensity to receive education being enhanced

We now examine what will happen to the economic system if the propensity to receive education is increased as follows: $\kappa_0 : 0.01 \Rightarrow 0.015$. We see that the lot size, agricultural land-use, and gold per capita are not affected. The effects on the other variables are plotted in Figure 3. The price of education is slightly increased. The time to receive education is augmented and the working time is reduced. The human capital is enhanced. The total labor supply falls initially and rises in the long term. In the short run, the time shifted to education reduces the labor supply, while the rise is human capital increases labor supply. The net effect reduces the labor supply in the short term. In the long term the net impact increases labor supply. The inputs and output level of the education sector are enhanced. The GDP and national physical wealth fall initially and rise in the long term. The prices of agricultural goods, land and gold and rents of land fall initially and rise in the long term. The output and capital and labor inputs of the two sectors are reduced initially and augmented in the long term. The wage rate falls and the rate of interest rises. The household's physical wealth and total wealth and consumption levels of two goods are lowered initially and are enhanced in the long term. We conclude that as the household increases the propensity to receive education, the household's economic conditions worsen initially and become improved in the long term.

Fig.3. The Propensity to Receive Education Being Enhanced



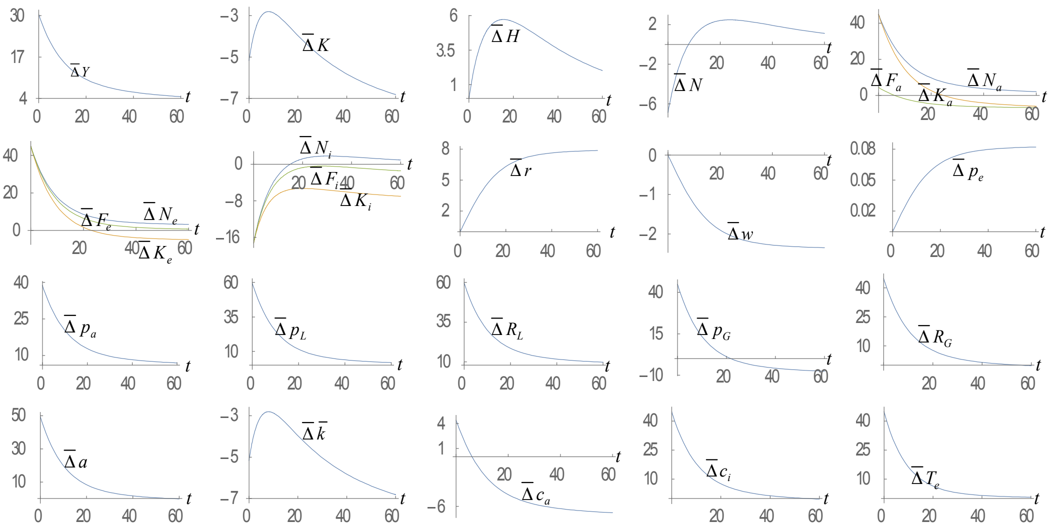
The propensity to consume housing being increased

We now consider what will happen to the economic dynamics if the propensity to consume housing rises as: The lot size and agricultural land-use change as follows

$$\overline{\Delta} l_h = 3.7, \ \overline{\Delta} L_a = -9.26.$$

Some land use is shifted from the agricultural use to the residential use. The rate of interest rises and the wage rate falls. The effects on the other variables are plotted in Figure 4. The price of agricultural goods rises and the price of education changes slightly. The land rent and land price rise. The gold price rises initially and falls in the long term. The gold rent rises and changes slightly in the long term. The household's physical wealth per capita falls. The household's total wealth rises initially and changes slightly in the long term. The consumption level of agricultural goods rises initially and falls in the long term. The household's consumption level of industrial goods rises initially and falls in the long term. The labor inputs of the education and agricultural sectors rise. The labor input of the industrial sector falls initially and rises in the long term. The capital inputs of the three sectors fall in the long term. The output level of the industrial (education) sector falls (rises). The output level of the agricultural sector rises initially and falls in the long term.

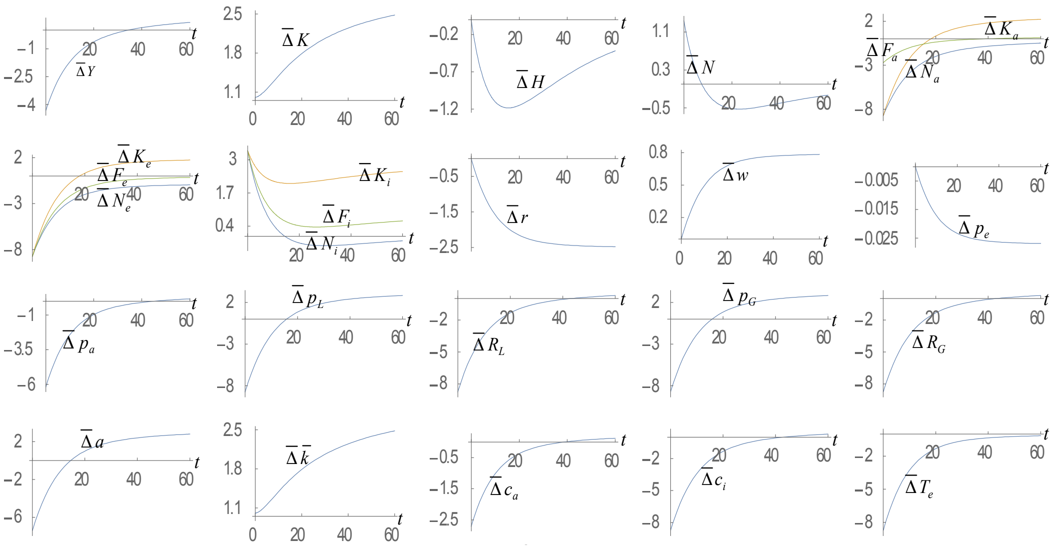
Fig.4. The Propensity to Consume Housing Being Increased



A rise in the propensity to save

Like almost any question in economic theory, one finds opposite answers in different theories. The Keynesian economic theory shows that a rise in the propensity to saving lowers national income, while the neoclassical growth theory suggests the opposite effect.

Fig.5. A Rise in the Propensity to Save

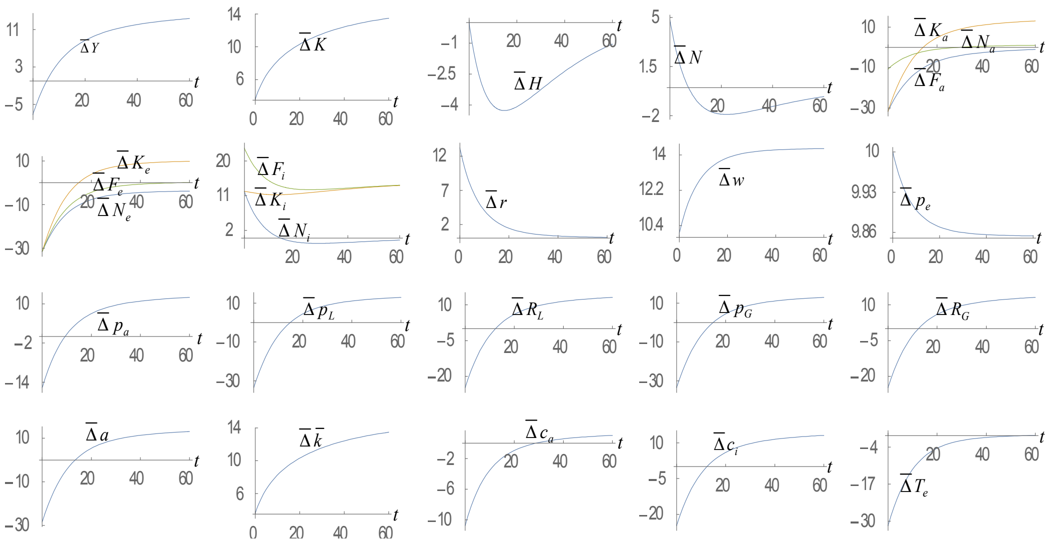


This sector allows the propensity to save to change as follows: $\lambda_0 : 0.8 \Rightarrow 0.82$. The change in the propensity has no impact on the lot size, agricultural land-use, and gold per capita. The rest of the simulation results are plotted in Figure 5. The physical wealth rises and the human capital falls. The national income is lowered initially and enhanced slightly in the long term. The wage rate rises and the rate of interest falls. The price of education falls. The price of agricultural goods falls initially and rises in the long term. The prices of land and gold fall initially and rise in the long term. The rents of land and gold fall initially and change slightly in the long term. The consumption levels of industrial goods and agricultural goods and education time fall and change slightly. The physical wealth per household is increased. The total wealth falls initially and is enhanced in the long term.

A rise in the total factor productivity of the industrial sector

We now study the effects of the following change in the total factor productivity of the industrial sector: $A_i : 1 \Rightarrow 1.1$. The effects on the time-dependent variables are plotted in Figure 6. The physical wealth rises. The human capital falls. The national income falls initially and rises slightly in the long term. The wage rate and the rate of interest rise. The labor force rises initially and falls in the long term. The education time falls and changes slightly. The price of education falls. The price of agricultural goods falls initially and rises in the long term. The prices and rents of land and gold fall initially and rise in the long term. The consumption levels of industrial goods and agricultural goods fall and change slightly. The physical wealth per household is increased. The total wealth falls initially and is enhanced in the long term.

Fig.6. A Rise in the Total Factor Productivity of the Industrial Sector



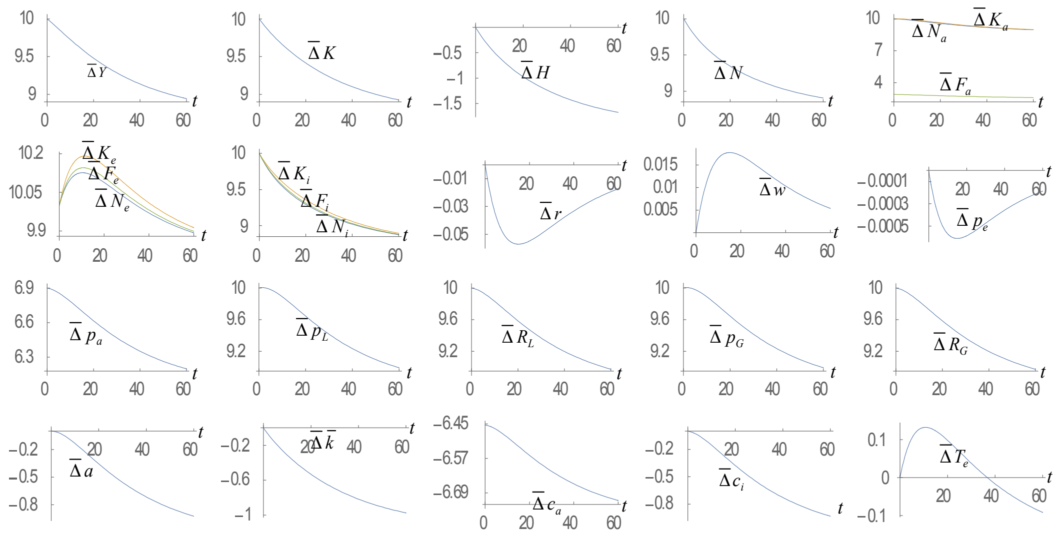
The population being augmented

The population rises as follows: $N_0 : 10 \Rightarrow 11$. The effects on the time-invariant variables are given as follows

$$\bar{\Delta} l_h = -9.09, \ \bar{\Delta} L_a = 0, \ \bar{\Delta} \bar{g} = -9.09.$$

The lot size and gold per capita fall. The agricultural land-use is not affected. The effects on the time-dependent variables are plotted in Figure 7. The rate of interest falls. The wage rate rises slightly. The prices and rents of land and gold are enhanced. The output levels and inputs of the three sectors are increased. The national output, total capital and total labor force are augmented. The human capital falls. The household initially spends more hours on education and reduces education time in the long term. The household's wealth levels and consumption levels of two goods are reduced. A larger population enhances the macroeconomic real variables and reduces the household's microeconomic real variables.

Fig.7. The Population Being Augmented



5. Concluding remarks

This study proposed a dynamic economic growth model of portfolio equilibrium between land, gold and physical wealth in a general equilibrium framework. The model examined dynamic interactions between land value, gold value, economic structural change, wealth accumulation and human capital growth by integrating the neoclassical growth theory, the Ricardian theory, and the Uzawa-Lucas model with Zhang's utility function. We modelled human capital dynamics by taking account of Arrow's learning by doing, Uzawa's learning through education, and Zhang's learning through consuming (leisure creativity). We simulated the motion of the economic system and demonstrated the existence of a unique stable equilibrium point. A comparative dynamic analysis was conducted with regard to changes in the propensity to consume gold, the propensity to receive education, the propensity to consume housing, the propensity to save, and the population.

We may extend and generalize the model in different directions. For instance, the model can be generalized by using more general function forms of the two sectors and the utility function. It is also possible to extend the model by taking account of heterogeneity of households. We may introduce some kind of government intervention in education into the model. In this study, we don't consider public provision or subsidy of education. In the literature on education and economic growth, many models with heterogeneous households are proposed to address issues related to taxation, education policy, distribution of income and wealth, and economic growth. The economy in our model has a fixed amount of gold owned by the households. Gold plays the role of storing value Gold can be sold in free markets without any friction and transaction costs. Both the assumption of fixed land and the assumption of fixed amount of gold are strict requirements (Barro, 1979; Barsky and Summers, 1988; Glaeser, *et al.*, 2005; Davis and Heathcote, 2007). Gold mining is an important industry and new supply brings about changes in gold markets. It is possible to introduce endogenous gold supply into our modelling.

Appendix: Proving the Lemma

The appendix shows that the dynamics can be expressed by two differential equations. From (3), (5) and (7), we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_a N_a}{K_a} = \frac{\tilde{\alpha}_e N_e}{K_e}, \qquad \text{(A1)}$$

where we omit time index and $\tilde{\alpha}_j \equiv \alpha_j / \beta_j, \ \ j = i, a, e.$ By (3) and (2), we have

$$r + \delta_k = \frac{\alpha_i A_i}{\tilde{\alpha}_i^{\beta_i}} z^{\beta_i}, \ \ w = \frac{\tilde{\alpha}_i^{\alpha_i} \beta_i A_i}{z^{\alpha_i}}, \qquad \text{(A2)}$$

where we also use (A1). We express w and r as functions of z.

From (15) and (19), we get

$$\mu \bar{y} \bar{N} = p_a F_a. \qquad \text{(A3)}$$

From (5), we have

$$r + \delta_k = \frac{\alpha_a p_a F_a}{K_a}. \qquad \text{(A4)}$$

From (A4) and (4) we solve

$$p_a \left(\frac{L_a}{K_a} \right)^{\varsigma} = \frac{\tilde{\alpha}_a^{\beta_a} (r + \delta_k)}{\alpha_a A_a z^{\beta_a}}, \qquad \text{(A5)}$$

where we use (A1). Equations (A3) and (A4) imply

$$\mu \bar{y} \bar{N} = \left(\frac{r + \delta_k}{\alpha_a} \right) K_a. \qquad \text{(A6)}$$

By (5) and (A3), we have

$$R_L = \frac{\varsigma \mu \bar{y} \bar{N}}{L_a}. \qquad \text{(A7)}$$

From $R l_h = \eta \bar{y}$ in (16) and (A7), we have

$$\varsigma \mu \bar{N} l_h = \eta L_a. \qquad \text{(A8)}$$

From (17) and (A8), we solve the land distribution as follows

$$L_a = \frac{\varsigma \mu L}{\eta + \varsigma \mu}, \ \ l_h = \frac{\eta L}{(\eta + \varsigma \mu) \bar{N}}. \qquad \text{(A9)}$$

The land distribution is invariant over time.

From (6) and (7) we have

$$p_e(z) = \frac{w z^{\alpha_e}}{\beta_e A_e \tilde{\alpha}_e^{\alpha_e}}. \tag{A10}$$

where we also use (A1). From the definition of $\bar{p}_e$ we have

$$\bar{p}_e(z, H) = p_e(z) + H^m w. \tag{A11}$$

From the definition of $\bar{y}$, we have

$$\bar{y} = (1 + r)\bar{k} + \left(1 + \frac{1}{r}\right) R_L \bar{l} + \left(1 + \frac{1}{r}\right) R_G \bar{g} + H^m T_0 w, \tag{A12}$$

where we use the definition of wealth and (8). Insert $R_L l_h = \eta \bar{y}$ and $R_G \hat{g} = \gamma \bar{y}$ in (15) in (A12)

$$\bar{y}(z, H, \bar{k}) = \omega_1 \bar{k} + \omega_2, \tag{A13}$$

where

$$\omega_0(z) \equiv \left[1 - \left(1 + \frac{1}{r}\right)\frac{\eta \bar{l}}{l_h} - \left(1 + \frac{1}{r}\right)\gamma\right]^{-1}, \quad \omega_1(z) \equiv \omega_0(1 + r), \quad \omega_2(z, H) \equiv \omega_0 T_0 H^m w.$$

Insert (A13) in $R_L l_h = \eta \bar{y}$ and $R_G \hat{g} = \gamma \bar{y}$

$$R_L(z, \bar{k}) = \frac{\eta \bar{y}}{l_h}, \quad R_G(z, \bar{k}) = \frac{\gamma \bar{y}}{\hat{g}}. \tag{A14}$$

From (15) and (A13) we have

$$T_e(z, H, \bar{k}) = \frac{\kappa \omega_1}{\bar{p}_e}\bar{k} + \frac{\kappa \omega_2}{\bar{p}_e}. \tag{A15}$$

From (A15) and (1) we get

$$N = T_0 H^m \bar{N} - \frac{\kappa \omega_2 H^m \bar{N}}{\bar{p}_e} - \frac{\kappa \omega_1 H^m \bar{N}}{\bar{p}_e}\bar{k}, \tag{A16}$$

$$N = p_1 - p_2 \bar{k}, \tag{A17}$$

where we also use the time constraint and

$$p_1(z, H) \equiv \left(T_0 - \frac{\kappa \omega_2}{\bar{p}_e}\right) H^m \bar{N}, \quad p_2(z, H) \equiv \frac{\kappa \omega_1 H^m \bar{N}}{\bar{p}_e}.$$

Insert (A1) in $N_i + N_a + N_e = N$

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_a}{\tilde{\alpha}_a} + \frac{K_e}{\tilde{\alpha}_e} = \frac{N}{z}. \tag{A18}$$

From (23) and (24) we have

$$K_i + K_a + K_e = \bar{k}\bar{N}. \tag{A19}$$

Inserting (A13) in (A6) we solve

$$K_a = \hat{\omega}_1 \bar{k} + \hat{\omega}_2 , \tag{A20}$$

where

$$\hat{\omega}_1(z, H) \equiv \left(\frac{\alpha_a}{r + \delta_k}\right) \mu \bar{N} \omega_1 , \ \ \hat{\omega}_2(z, H) \equiv \left(\frac{\alpha_a}{r + \delta_k}\right) \mu \bar{N} \omega_2 .$$

Insert (A20) and (A17) in (A18) and (A19)

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_e}{\tilde{\alpha}_e} = b_1\left(z, H, \bar{k}\right) \equiv \bar{p}_1 - \bar{p}_2 \bar{k} , \tag{A21}$$

$$K_i + K_e = b_2\left(z, H, \bar{k}\right) \equiv \bar{p}_0 \bar{k} - \hat{\omega}_2 ,$$

where

$$\bar{p}_1(z, H) \equiv \frac{p_1}{z} - \frac{\hat{\omega}_2}{\tilde{\alpha}_a}, \ \ \bar{p}_2(z, H) \equiv \frac{p_2}{z} + \frac{\hat{\omega}_1}{\tilde{\alpha}_a}, \ \ \bar{p}_0(z, H) \equiv \bar{N} - \hat{\omega}_1 .$$

Solve (A21)

$$K_i = \alpha_0 \left(b_1 - \frac{b_2}{\tilde{\alpha}_e}\right), \ \ K_e = \alpha_0 \left(\frac{b_2}{\tilde{\alpha}_i} - b_1\right), \tag{A22}$$

where

$$\alpha_0 = \left(\frac{1}{\tilde{\alpha}_i} - \frac{1}{\tilde{\alpha}_e}\right)^{-1} .$$

Insert the definitions of b_j in (A21) in (A22)

$$K_i = m_i \bar{k} + \bar{m}_i , \ \ K_e = m_e \bar{k} - \bar{m}_e , \tag{A23}$$

where

$$m_i(z, H) \equiv - \alpha_0 \left(\bar{p}_2 + \frac{\bar{p}_0}{\tilde{\alpha}_e}\right), \ \ \bar{m}_i(z, H) \equiv \alpha_0 \left(\bar{p}_1 + \frac{\hat{\omega}_2}{\tilde{\alpha}_e}\right),$$

$$m_e(z, H) \equiv \alpha_0 \left(\frac{\bar{p}_0}{\tilde{\alpha}_i} + \bar{p}_2\right), \ \ \bar{m}_e(z, H) \equiv \alpha_0 \left(\frac{\hat{\omega}_2}{\tilde{\alpha}_i} + \bar{p}_1\right).$$

We solved the capital distribution as functions of z, H and $\bar{k}$. By (A1), we solve the labor distribution as functions of z, H and $\bar{k}$ as follows

$$N_i = \frac{z K_i}{\tilde{\alpha}_i}, \ \ N_a = \frac{z K_a}{\tilde{\alpha}_a}, \ \ N_e = \frac{z K_e}{\tilde{\alpha}_e}. \tag{A24}$$

From (6) and (18) we have

$$T_e \bar{N} = K_e A_e \left(\frac{z}{\tilde{\alpha}_e}\right)^{\beta_e} , \tag{A25}$$

where we also use (A1). Insert (A23) and (A15) in (A25)

$$\bar{k}(z, H) = \left[\frac{\kappa\omega_2 \overline{N}}{\bar{p}_e} + \left(\frac{z}{\tilde{\alpha}_e}\right)^{\beta_e} A_e \overline{m}_e\right]\left[\left(\frac{z}{\tilde{\alpha}_e}\right)^{\beta_e} A_e m_e - \frac{\kappa\omega_1 \overline{N}}{\bar{p}_e}\right]^{-1}. \tag{A26}$$

From (8) and (9) we have

$$a = \phi(z, H) \equiv \bar{k} + p_L \bar{l} + p_G \bar{g}. \tag{A27}$$

It is straightforward to check that all the variables can be expressed as functions of z and H at any point in time as follows: r and w by (A2) → $\bar{k}$ by (A27) → L_a and l_h by (A9) → p_e by (A10) → $\bar{p}_e$ by (A11) → p_a by (A5) → $\bar{y}$ by (A13) → R_L and R_G by (A14) → p_L and p_G by (A8) → K_i and K_e by (A23) → K_a by (A20) → N_i, N_e, and N_a by (A1) → N by (A17) → a by (A27) → F_i by (2) → F_a by (4) → F_e by (6) → c_a by (19) T_e, c_i, and s by (15) → T by (13) → C_a by (19) → $C_i = c_i \overline{N}$ → $\bar{l}$ by (20) → $\bar{g}$ by (21). From this procedure, (16) and (17), we have

$$\dot{a} = \Lambda_0(z, H) \equiv s - a, \tag{A28}$$

$$\dot{H} = \Omega(z, H) \equiv \frac{\upsilon_e F_e^{a_e}\left(H^m T_e \overline{N}\right)^{b_e}}{H^{\pi_e} \overline{N}} + \frac{\upsilon F_i^{a_i}}{H^{\pi_i} \overline{N}} + \frac{\upsilon_h C_i^{a_h}}{H^{\pi_h} \overline{N}} - \delta_h H. \tag{A29}$$

Taking derivatives of (A27) with respect to t yields

$$\dot{a} = \frac{\partial \phi}{\partial z}\dot{z} + \Omega\frac{\partial \phi}{\partial H}, \tag{A30}$$

in which we use (A29). Equal (A28) and (A30)

$$\dot{z} = \Lambda(z, H) \equiv \left(\Lambda_0 - \Omega\frac{\partial \phi}{\partial H}\right)\left(\frac{\partial \phi}{\partial z}\right)^{-1}. \tag{A31}$$

From (A29) and (A31), we determine the motion of z and H. We thus proved the lemma.

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