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Spatial Threat Level Analysis

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Abstract

In the material the authors consider methods for the analysis of spatially-defined states, which are an indicator of certain threats. One such threat is the evolving COVID-19 epidemic. An essential part of the countermeasures, especially in the early stages, is the warning not to enter areas with an increased risk of infection, or in the case of partially quarantined persons - a warning not to leave the perimeter approved for movement. Modern mobile devices, smartphones, are a natural tool for personal alerting a person when entering a territorial area with undesirable consequences. Traditional methods for localization are considered, as well as a method for determining the coordinates of a mobile object using network stations of mobile operators. The mathematical bases are presented and an analysis is performed to evaluate the effect of the spatial analysis.

Keywords: cellular network, navigation, awareness

JEL: C61

1. Introduction

In everyday life, as well as in a large part of the work activity, people move along set, or sometimes approximately certain routes. It is normal, at least within the state borders, for the movement not to be territorially limited and to be determined by personal preferences or according to the economic activity. The greatly changed public environment since the declared COVID 19 pandemic has led to the introduction of a number of measures, which in extreme cases are related to restrictions on movement. As a rule, restrictions are declared as administrative measures and cover countries, districts, cities and in some cases even individual neighborhoods. On the other hand, a person could be warned of the danger of entering an area of increased epidemic danger if he or she has appropriate personal equipment. Such a device could be your own smartphone, with the appropriate software application.

Two main tasks need to be solved in order to trigger a spatial warning. The first is to introduce danger zones. This is a responsible task and can be difficult or even impossible for the average person to perform on their own smartphone. A state-level information infrastructure should be created with a software interface to automatically retrieve the contours of high-risk areas.

The next task is to conduct an analysis of the spatial state of the moving object. The analysis is subject to full automation and is mathematically provided in its computational part. Constantly, during a given time interval, the current coordinates of the moving object, location are determined. The current

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coordinates obtained during the localization are analyzed for falling inside the contour of one of the danger zones. If the result is positive, a warning signal is issued.

In the present paper, the authors review mathematical methods for performing spatial analysis for positioning in a hazardous area and present their numerical results regarding the applicability of these methods in urban settings.

2. Methods for localization in urban conditions

2.1. Built-in GPS receiver

The Global Positioning System (GPS) is a satellite-based system designed to determine the position of a GPS receiver in 3D coordinates, speed and astronomical time at any time of the day, in all weather conditions and anywhere in the world. The position of the GPS receiver is determined by the mathematical method of spherical trilateration. The modulated signal from the GPS system allows determining the distance between the GPS receiver and a visible GPS satellite. This distance determines the sphere with the center of the satellite on which sphere the receiver should be located. The position of the satellite in outer space is also transmitted to the receiver. If three satellites are used, three corresponding spheres will intersect at the location of the receiver. In practice, an additional satellite is needed to compensate for the time difference between the receiver's clock and the operating time of the GPS system.

The advantages of the GPS location system are sufficient for practical needs, accuracy and accessibility of the technology. However, serious shortcomings limit its mass application for urbanization. Not all cell phones in use have a built-in GPS receiver. Ideally, the localization system should be compatible with the most common cell phones currently in use. Another inherent disadvantage of the GPS system is the Line of Sight (LOS) limitation. The modulated GPS signal requires direct visibility on the road between the receiver and the satellites and this makes GPS poorly usable inside buildings, vehicles, underground car parks, tunnels and other areas that are considered critical in terms of national security and defense tasks.

2.2. Triangulation

Triangulation is one of the most commonly used techniques to determine the position of a radio transmitting object. Although there are several triangulation techniques in general, they use similar methods (Ibrahim et al., 2010). Measurements shall be made at two or more points, and a line at the angle of maximum signal strength shall be determined for each point. The point where the lines intersect is the position of the radio transmitting object (Figure 1).

In the example, two-point triangulation is used. Assume that the angles of the maximum signal strength line are measured at two points with known coordinates $M_1(x_1, y_1)$ and $M_2(x_2, y_2)$, as shown in Figure 1, and it is necessary to find the coordinates of the intersection point$P(x_p, y_p)$.

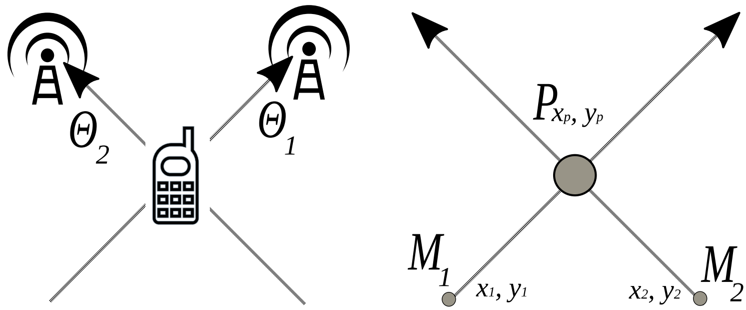

Figure 1. Triangulation technique

The equation of lines through the point M1 (x1, y1) is:

$$y - y_1 = m(x - x_1), \tag{1}$$

or

$$y = mx - mx_1 + y_1 \tag{2}$$

Then the equation of the line M1 - P is:

$$y = \tan\theta_1 x - x_1 \tan\theta_1 + y_1 \tag{3}$$

Similarly, the equation of the line M2 - P is:

$$y = \tan\theta_2 x - x_2 \tan\theta_2 + y_2 \tag{4}$$

We find the intersection point x_p as:

$$x_p = \frac{(x_1 \tan\theta_1 - y_1) + (y_2 - x_2 \tan\theta_2)}{\tan\theta_1 - \tan\theta_2} \tag{5}$$

$$y_p = \tan\Theta_2 x_p - \tan\Theta_2 x_2 + y_2 \tag{6}$$

One of the key prerequisites for successful application of the triangulation method for mobile object localization is the ability to use a cell phone to accurately determine the angle of the line with the strongest signal to two or more base stations in the cell network. In addition, the fact that the localization procedure does not require special actions of the cell phone user further complicates the application of this method.

The analysis of the principles of operation, advantages and disadvantages of the localization methods with built-in GPS receiver and using triangulation to known base stations of the cellular network shows that both traditional methods are not sufficiently applicable in the task conditions: urbanization with use of widely available devices such as a low-end cell phone.

2.3. Trilateration

The mathematical method of spherical trilateration for object localization is applicable when at least three base points with known coordinates and distances (r1, r2, r3) from the object to them are available (Figure 2). When the site is a cellular telephone for base points, it is natural to choose the nearest base stations on the cellular network.

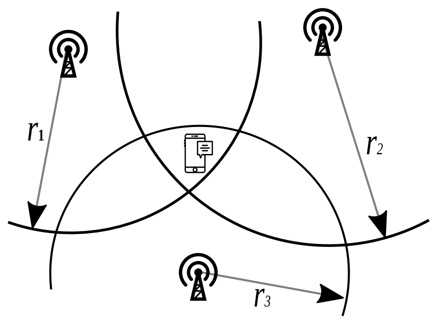

Figure 2. Trilateration as a method of localization

The specifications of the most used cellular network (GSM-Global System for Mobile Communications) require at any time during the operation of the cell phone the system to provide information about the identifiers of three available base cells (Cell ID), the geographical area of the cells Location Area Code (LAC) and the received signal strength (RSS) to each of these cells.

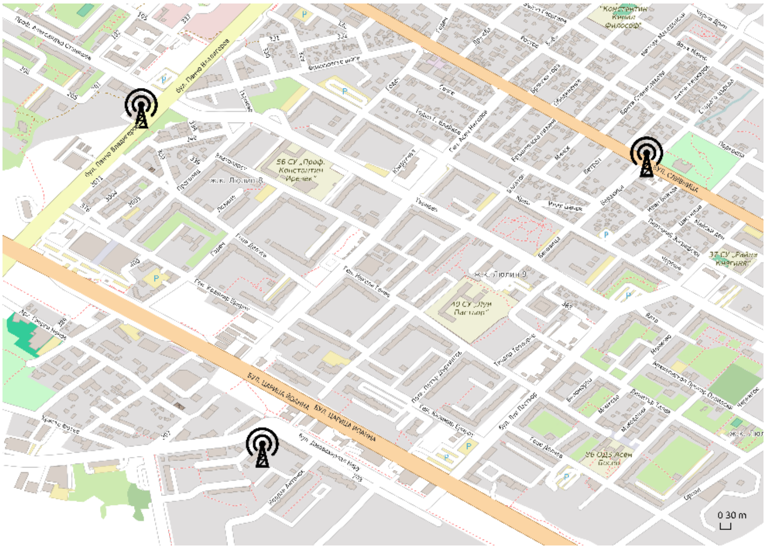

Figure 3. Graphical representation of OpenCellID

Official information on the coordinates of the base cells of a given cellular network has not been published. However, from a practical point of view, several organizations exist and are in the process of filling in and improving the accuracy of the data, which maintain databases on cellular network base stations. OpenCellID, CellDB and CellSpotting are known as such. The OpenCellID organization offers an Application Program Interface (API), with the help of which the geographical coordinates of a base station from a cellular network are accessed by requested LAC and Cell ID. In Figure 3 are presented in graphical form part of the published data of OpenCellID for a highly populated area on the outskirts of Sofia.

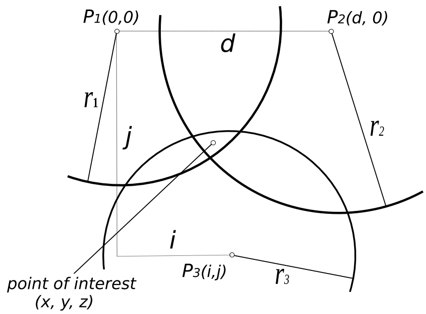

Figure 4. Graphic design for trilateration

We assume that at some point in time the cell phone object has data on the geographical coordinates of three available base stations in a metric measurement system. The distance to each of the base stations is to be determined. It is possible to use the methods of Time of Arrival (TOA), Time Difference of Arrival (TDOA), Received Signal Strength (RSS) to determine the distance to the base cell. According to the literature, the application of the above methods allows the determination of a distance with an accuracy of 5 to 70 meters (Smith et al., 2006).

The mathematical approach of trilateration is explained in Figure 4.

The intersection point of the surfaces of three spheres (Manzoni et al., 2019) is formulated as a system of equations that are solved with respect to the sought point (x, y, z). To simplify the mathematical representation, we assume that the centers of the spheres lie in one plane z = 0.

Equations of the three spheres (Erik Mahieu 2015):

$$\left.\begin{aligned} r_1^2 &= x^2 + y^2 + z^2 \\ r_2^2 &= (x-d)^2 + y^2 + z^2 \\ r_3^2 &= (x-i)^2 + (y-i)^2 + z^2 \end{aligned}\right\} \tag{7}$$

It is necessary to find a point (x, y, z) as the intersection of the three spheres:

$$\left.\begin{aligned} x &= \frac{r_1^2 - r_2^2 + d^2}{2d} \\ y &= \frac{i{r_2}^2 - d{r_3}^2 - (i-d){r_1}^2 + dj^2 + di^2 - d^2 i}{2dj} \\ z &= \pm\sqrt{r_1^2 - x^2 - y^2} \end{aligned}\right\} \tag{8}$$

Which is possible if the condition is met

$$d - r_1 < r_2 < d + r_1. \tag{9}$$

In cases where only a planar solution is considered (Mounir et al., 2017), the matrix representation of mathematical dependence has the form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2(x_1 - x_3) & 2(y_1 - y_3) \\ 2(x_2 - x_3) & 2(y_2 - y_3) \end{bmatrix}^{-1} \begin{bmatrix} x_1^2 - x_3^2 + y_1^2 - y_3^2 + r_3^2 - r_1^2 \\ x_2^2 - x_3^2 + y_2^2 - y_3^2 + r_3^2 - r_1^2 \end{bmatrix} \tag{10}$$

The sequence for information interaction at localization is:

- The mobile object as a cellular phone interacts with the GSM network and determines the CellID, RSS and LAC of three accessible base stations of the cellular network
- The cellular phone establishes a General Packet Radio Service (GPRS) or other wireless connection to a system server and transmits Cell ID, RSS and LAC parameters to the network environment;
- The system server uses the OpenCellID API and determines the geographical coordinates of the base stations of the network environment;
- The system server applies the RSS data, determines the r1, r2, and r3 distances to the base stations and determines the geographical coordinates of the cell phone using the expressions (8, 9, 10);
- The system server represents the localization at a higher hierarchical level in the unified environment for managing the epidemic crisis.

3. Assessment of the probable deviation in localization

The variable factor that affects the correct localization of the mobile object is the inability to know the exact distances to the base stations of the cellular network, denoted above by r1, r2, and r3. According to the literature cited above, the inaccuracy in determining these distances in an urban environment reaches several tens of meters. In order to determine the degree of practical applicability of the presented trilateral method of localization, the authors present their numerical assessment of the probable deviation in determining the location.

A general purpose Computer Aided Design (CAD) software product obtained under a free license is used for analysis. The locations of three actually existing base stations from the cellular network of a GSM operator are introduced in a relatively positioned metric coordinate system. The mean distance between base cells is approximately 1 km (1068.6 m), generally shown in Figure 5. The independent parameter that takes into account the inaccuracy in determining the distance from the moving object to any of the base cell stations is indicated by *Rerr*. The analysis was performed by introducing two limit values for *Rerr*, for which the values of 5 m and 70 m were adopted.

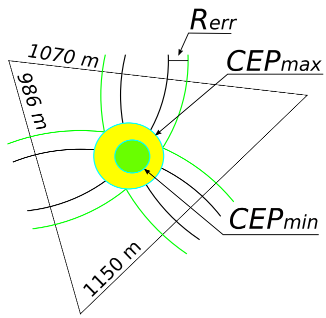

Figure 5. Estimation of the localization error

The resulting inaccuracy in the localization of the moving object is specified in terms of "Circular Error Probable (CEP)" (Webb, 2012) and are taken into account with a minimum error value, CEPmin and its maximum value, CEPmax.

The exact numerical values obtained as a result of the analysis are presented in Table 1.

Table 1. Localization Circular Error Probable values

Rerr **[m]**	***CEP*** **[m]**	
	min	***max***
5	4.9	9.9
70	62.6	124.4

4. Spatial analysis

An acceptable mathematical approach is to solve the problem known from the computational geometry "point within the polygon", where the polygon is the probable danger zone, and the point for which access is sought is the coordinates of the location of the moving object.

The calculation problem "point in a polygon" can be solved by dividing elementary triangles, by the number of intersection points of the polygon with lines through the position of the object (ray casting algorithm), or by the value of the central angle. Let us take a closer look at the method of dividing elementary triangles.

After the polygon is divided into triangles, we check whether the point in question is within the limits of any of the resulting triangles (Figure 6).

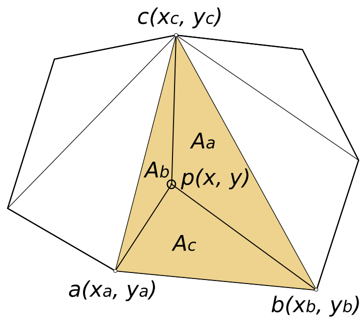

Figure 6. Graphical computational problem

Let us assume that the studied elementary triangle is defined by its vertices a (x_a, y_a), b (x_b, y_b) and c (x_c, y_c). The test point is denoted by p (x, y). By applying the barycentric coordinate method (Weisstein, 2015; Shirley et al., 2009), we test the point p to see if it falls within Δabc.

We define the area of the triangle Δabc as a scalar product of dependence vectors:

$$A = \frac{1}{2}(\overrightarrow{ab} \ . \ \overrightarrow{ac})$$

$$A = \frac{1}{2}\begin{vmatrix} x_b - x_a & x_c - x_a \\ y_b - y_a & y_c - y_a \end{vmatrix} \qquad (11)$$

Or:

$$A = \frac{1}{2}(x_a y_b + x_b y_c + x_c y_a - x_a y_c - x_b y_a - x_c y_b) \tag{12}$$

The barycentric coordinates α, β and γ are defined as:

$$\left.\begin{aligned} \alpha &= \frac{A_a}{A} \\ \beta &= \frac{A_b}{A} \\ \gamma &= \frac{A_c}{A} \end{aligned}\right\} \tag{13}$$

Then

$$\left.\begin{aligned} \gamma &= \frac{(y_a - y_b)x + (x_b - x_a)y + x_a y_b - x_b y_a}{(y_a - y_b)x_c + (x_b - x_a)y_c + x_a y_b - x_b y_a} \\ \beta &= \frac{(y_a - y_c)x + (x_c - x_a)y + x_a y_c - x_c y_a}{(y_a - y_c)x_b + (x_c - x_a)y_b + x_a y_c - x_c y_a} \\ \alpha &= 1 - \beta - \gamma \end{aligned}\right\} \tag{14}$$

The point p falls in the triangle Δabc when the conditions are simultaneously fulfilled:

$$\left.\begin{aligned} 0 < \alpha < 1 \\ 0 < \beta < 1 \\ 0 < \gamma < 1 \end{aligned}\right\} \tag{15}$$

When the calculation problem “point in a polygon”, solved by one of the above mathematical methods does not give a solution, we have the case of a moving object outside the danger zone. Otherwise, a violation signal is given and the person is warned about the occurrence of an undesirable situation.

Conclusions

Restrictions on entering geographically defined areas are not typical of our time, but they have become necessary in the scope of anti-epidemic measures in the fight against COVID 19. Administrative restrictive measures are effective in themselves. Having a personal warning system that responds to unintentional entry into an area declared dangerous would be a useful opportunity to calm an already exciting day.

From the methods discussed in the material for localization in urban conditions and determination of spatial affiliation, it is clear that a suitable mathematical apparatus is available. The trilateration method based on identified cellular network base stations is workable and applicable for localization of mobile objects equipped with smartphones, even of the lowest user class.

The practical applicability of the method is directly related to the accuracy of determining the distances between the site and the nearest basement cell stations. The discussed possibility to determine the distances in question as a function of the strength of the received RSS signal is associated with difficulties of a purely physical nature. In this regard, in urban conditions, the attenuation of the signal due to the absorption of the material of the opposing buildings, attenuation (fading) under conditions of multipath propagation have a disturbing effect.

These factors are subject to mathematical modeling, specialized software products are known and are used in the professional systems of GSM operators. A low-budget solution would be possible by using adjustments based on statistical data and in relation to the types of development in the specific urban environment.

The localization error when applying the discussed method of trilateration in urban conditions in the worst case is expected to be of the order of 100 meters. Considered as a next step, the creation of own test application for a mobile device will allow experimental refinement of the localization error numerical data, compared to similar functionality of the standard Android operating system applications.

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