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Ambiguous Strategy Choice in an Economic Game D(2,2) Describing a Competition

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Georgi Kiranchev *

Abstract

This study examines in detail the game known as the “two-bar dilemma” in the general form of the payoff matrices of the two player-direct competitors. This dilemma is exclusively in the field of economics and competition. Its familiarity is therefore useful for readers who are concerned with decision making in economics in a competitive environment.

All conclusions are the result of mathematical proofs and not of specific numbers specifically selected for this purpose and therefore the conclusions are generalized over all possible variants of payment matrices.

The game has been shown to be much more complex and rich in possibilities than the widely known “prisoner’s dilemma”, to which the “two-bar dilemma” game was wrongly considered a complete analogue.

It is explained why these two games cannot be analogous and why the game “dilemma of the two bars” is a more complex case.

It has been shown that it is possible to have two Nash equilibria, which is not possible in the Prisoner’s Dilemma.

It is proved that there is no optimal mixed strategy for any of the players in this game and that the players have only optimal pure strategies, different in different situations.

The difference between unit profit and unit marketing cost has been shown to be important for the players’ choice of pure strategy, but it is not the unique determinant.

It has been shown that a Nash equilibrium formed by pairs of “asymmetric” strategies cannot exist.

It has been shown that the “loyalty” of the customers of the two bars does not matter for the choice of an optimal strategy.

Keywords: Game Theory, Nash Equilibrium, dilemma, marketing, strategy, promotion

JEL: C72, M31, M37

Description of the dilemma of the two bars

Our purpose is not to discuss and analyze the “prisoner’s dilemma”, which is a much simpler game, but another situation, not as widely known, which is also described as D (2,2) and which is mistakenly considered to be a complete economic analogue of the prisoner’s dilemma. This situation is usually referred to as the “two-bar dilemma”, simply “the two bars”, but can be found under other names as well.

The classic statement of the dilemma of the two bars is as follows:

There are two bars (two players), A and B, which are direct competitors and they are

* University of National and World Economy - Marketing and Strategic Planning

fighting for the customer contingent. There are reasons to believe that if a happy hour is announced during which a free serving of peanuts is served with a purchased beer, this will attract some part of the customers of the competing bar and lead to the following results:

If one bar owner, regardless of which of the two bar owners, introduces a happy hour in his bar, he will attract some of the competitor's customers, and the competitor will lose them and therefore reduce his profits.

If neither of the two introduces a happy hour, the situation will remain as it is actually.

If both owners introduce a happy hour, no one will attract additional customers, but because of the additional cost - the free peanuts, each of them will see their profits decrease.

Each of the owners has to choose whether or not to introduce a happy hour and try to take away some of the competitor's customers. But since they are competitors and do not bargain with each other, no one knows how the other will act. How will the two act?

Different formulations may encounter different size payoffs, but they are always subject to certain dependencies, as in the prisoner's dilemma, so the decision is always unambiguous.

i. Payment matrix in the "two bars" game (example)

	B1: no happy hour	B2: happy hour is introduced
A1: no happy hour	10; 10	8; 11
A2: happy hour is introduced	11; 8	9; 9

In this case, gains are recorded in the payment matrix and therefore all dominations will be of the "**more is better**" type.

It can be seen that in this case each of the two competing players has a strictly dominant strategy - to introduce a happy hour and try to increase their profit at the expense of some of the competitor's customers:

$$\begin{matrix} 11 \\ 9 \end{matrix} \succ \begin{matrix} 10 \\ 8 \end{matrix}$$

As a result, they will both introduce a happy hour, where the profits of both will not increase but decrease, after all, free peanuts for customers are not free for bar owners either. Customers get profit.

On the basis of the numbers (or similar numbers) so chosen, the conclusion is that this game is not a separate, economic game, with its own peculiarities, but a complete analogue of the prisoner's dilemma. On the basis of such examples, the 'two bars' game is everywhere referred to as a case of the 'prisoner's dilemma' and false conclusions are drawn about the role of marketing costs.

Our research is aimed at a more in-depth analysis of the two-bar game. Rather than cherry-picking some convenient numbers, all inferences will be made on the basis of payment matrices in the general case, without artificially imposing any convenient constraints on payments. The author's opinion is that no examples with individual numbers have a probative value, unlike mathematical proof in the general case.

We will prove that the "two bars" game is much more complicated than the "prisoner's dilemma" and that it has no simple and unambiguous solution in the general case, but that the solution depends on certain conditions that describe an economic situation. Our statement is that this game is radically different from the prisoner's dilemma and that it is incorrect to consider it as its analogue, but rather that the prisoner's dilemma is a special case of this game.

We will derive and point out the conditions under which the game has a unique solution and the conditions under which it does not.

This will be done by introducing a minimal number of parameters that describe the "two-bar" game more realistically. Rather than come up with some numbers on which convenient constraints are imposed, we will introduce a reasonable minimum of parameters and now depending on these we will explore the payment matrix problem in general.

We will point out right here that there is a fundamental difference between the prisoner's dilemma situation and the two-bar situation. Although both are described as D (2,2), in the case of the prisoner's dilemma, a third **rational** force, the country's judicial system, has made efforts to create and maintain the necessary conditions for the two players to have dominant strategies and for their actions to be what is beneficial to society. The judiciary is supposed to be reasonable and pursue its own particular goals, one of which is to punish criminals and, if possible, everyone.

That is not how things are in the two-bar game.

First of all, there is no system that organizes the game for any purpose. All that is required is that the two players do not collude with each other, which means that the game does not become cooperative.

Secondly, no one cares to create and maintain the conditions for the existence of dominant strategies of each of the two players. Competition is entrusted with the functions of game organiser, but it is not a rational force and does not pursue any objectives.

Thirdly, the parameters of the game determining the values in the payment matrix may differ for each of the players, and generally they are what has emerged in the process of the competitive struggle between the two bars (the two players) for their share of the customers.

Modelling the situation with the two bars

Here we model the situation that results in the payment matrices of the two players.

For this purpose, we will introduce the following parameters describing the situation in the general case:

S_1 - number of customers at the first bar;

S_2 - number of customers at the second bar;

p - profit on the sale of one beer;

m - "marketing costs" - the price of one serving of peanuts with the ordered beer;

q, $(0<q<1)$- the portion of customers who are willing to take the opportunity to get free peanuts with their beer (happy hour), even if they have to change bars to do so.

Additional logical assumptions are:

In general, the number of customers of the two bars is not equal. The equal number of customers is only a particular case, from which no particular and different conclusions and results follow.

The total number of customers is $S_1 + S_2$ and under the influence of marketing costs they can be redistributed between the two bars, but their sum remains constant. Otherwise an additional set of potential additional customers must be introduced. This assumption is realistic in some time interval where the demographics of the area do not change significantly, i.e. the population that goes to a bar due to its age does not increase or decrease. After long enough, this situation could change.

None of the competitors have access to free beer or free peanuts. This means that free peanuts for customers are not free for the bar owner and represent an additional cost to him.

Beer prices are competitive, i.e. no one makes excess profit and the margin for both bars is the same. Otherwise, one of the two players could, for example, give away many more peanuts for free than the other or top up an extra amount of beer.

Only some customers will take advantage of the free peanuts with the beer at the advertised happy hour for a variety of reasons. Those willing to do so will do so regardless of whether the peanuts are offered at "their" bar or the other. Conversely, people who won't take advantage of a happy hour won't do it regardless of whether the happy hour is at "their" bar or the other.

One may say that the part $0<(1-q)<1$ is a measure of customer loyalty with respect to "their" bar, but that wouldn't be accurate. Here we will list some of the reasons why some customers will not change their bar, which on the surface would look like "loyalty". The list does not pretend to be exhaustive and can certainly be extended by professionals researching the topic of "loyalty".

1. the extra serving of peanuts may not appear appealing to some customers because they are not drinking their beer with peanuts but with, for example, potato chips or anything other than peanuts (the attracting force does not work);
2. the extra serving of peanuts is only served when ordering 0.5l beer, but not for smaller orders of 0.33l or 0.25l. This assumption may seem artificial, but as we will see later, there is a relationship between the profit per beer and the cost of peanuts. We will refer to this again when we draw conclusions about what conditions the profits and costs of free peanuts should satisfy (the attracting force does not work because it is related to a violation of own preferences);
3. the happy hour is inconvenient for some customers and therefore they will not take advantage even though they would at another time (attracting force does not work at all);
4. the other bar is further away and some customers will not go the extra and further just for a serving of peanuts (attracting force is weak);
5. the environment at the other bar does not appeal to customers who would benefit from a happy hour (there is a stronger repulsive force at the other bar);
6. the beloved company likes this bar and will not understand a member leaving it for a serving of peanuts (there is a stronger attracting force in "their" bar);
7. the customer would benefit from a happy hour, but the girlfriend/boyfriend likes this bar and doesn't want to change (there is a stronger attracting force in "his" bar);
8. they watch their favorite sports channel in 'their' bar and the music channel in the other bar (there is a stronger attracting force in "their" bar and a repulsive force in the other bar);
9. in the other bar the customer's unpleasant company gathers (there is a repulsive force greater than the attracting force);

As can be seen, customers are loyal primarily to themselves, to their habits, tastes and preferences, and to their comfort. And, if they have to be wronged for some portion of peanuts, they will not do it. On the surface, this looks like loyalty in terms of the bar, the "brand," the "brand name," or whatever else appeals to the "loyalty" researcher who is inclined to declare them "100% loyal." But in fact it's another loyalty - in relation to themselves, the most beloved.

Under these conditions and notations, we will form in general terms the payment matrices of the two players. For player A:

ii. Player A's payment matrix

Player A/ Player B	B1: Does not give free peanuts	B2: Gives free peanuts
A1: Does not give free peanuts	S_1p	$S_1(1\text{-}q)p$
A2: Gives free peanuts	$S_1q(p\text{-}m) + S_2q(p\text{-}m) + S_1(1\text{-}q)p$	$S_1q(p\text{-}m) + S_1(1\text{-}q)p$

Explanation of the expressions in the payment matrix:

The strategy pair ***{Not giving free peanuts; Not giving free peanuts }***: if both players do not give free peanuts, they keep their number of customers and their profit from a glass of beer, which in player A's case is S_1p. This is the case in which there is no redistribution of customers.

The pair of strategies ***{Not giving free peanuts; Gives free peanuts}***: if player A does not give free peanuts and player B does the opposite, then player A will lose *q* part of his customers and his profit $S_1(1\text{-}q)p$ will decrease at the expense of lost customers.

The strategy pair ***{Gives free peanuts; Not giving free peanuts }*** contains 3 elements that require explanation each separately:

$S_1q(p\text{-}m)$ is the gain (possibly – the loss) to player A from his customers getting a free glass of beer with free peanuts. The profit from the glass of beer is reduced by the price of the peanuts;

$S_2q(p\text{-}m)$ is the profit (possibly - the loss) for player A from the attracted customers of the other bar who will change the bar to get a glass of beer and free peanuts. The profit from the glass of beer is also reduced by the price of the peanuts;

$S_1(1\text{-}q)p$ is the usual profit from those customers who will not take advantage of a happy hour and will get their beer outside of it. It is not reduced by the price of peanuts, but these are only a part of the customers at the first bar.

The first two elements $S_1q(p\text{-}m)$ and $S_2q(p\text{-}m)$ separately describe the profit from customers who will take advantage of a happy hour, and the third element describes the profit from those (own customers) who will not take advantage of a happy hour.

The pair of strategies ***{Gives free peanuts; Gives free peanuts}***: if both players give free peanuts, they keep their number of customers, but their profit decreases at the expense of the part of customers who will order a glass of beer at a happy hour and get free peanuts. In the expression there are no customers attracted from the other bar because they will stay there. This profit represents a weighted average of the profits from customers who will benefit from a happy hour and those who will not.

The payoff matrix of player B is described similarly:

iii. Player B's payment matrix

Player A/ Player B	B1: Does not give free peanuts	B2: Gives free peanuts
A1: Does not give free peanuts	S_2p	$S_1q(p\text{-}m)+ S_2q(p\text{-}m)+ S_2(1\text{-}q)p$
A2: Gives free peanuts	$S_2(1\text{-}q)p$	$S_2q(p\text{-}m)+ S_2(1\text{-}q)p$

So far there are no discrepancies with the regular game description, only the numbers are replaced with expressions in general form.

As one can see, there is some symmetry in the payoffs of the two players with respect to the main diagonal of the payoff matrix, so the conclusions obtained for one player will be true for the other "by symmetry".

Let us investigate the different situations and conditions for the existence of Nash Equilibrium (henceforth denoted as NE for short) in pure strategies, for their absence and for dominance.

Condition that the strategy of not incurring additional marketing costs is dominant for both players

This condition is interesting in that it sets a more than sufficient condition for the existence of a unique a single NE, which will be formed by the strategy pair ***{No free peanuts; No free peanuts}***. In order for the strategy of no additional marketing costs to be the dominant strategy for both players, four conditions must be satisfied.

For player A, the strict dominance condition is:

$$\left| \begin{array}{c} S_1 p > S_1 q(p-m) + S_2 q(p-m) + S_1(1-q)p \\ S_1(1-q)p > S_1 q(p-m) + S_1(1-q)p \end{array} \right.$$

After simplifying the expressions we get:

$$\left| \begin{array}{c} 0 > S_1 q(p-m) + S_2 q(p-m) - S_1 qp \\ 0 > S_1 q(p-m) \end{array} \right.$$

Since by the default condition *0<q<1*, we can truncate this multiplier in both inequalities without causing a change in the directions of the inequalities and without performing division by zero and we get the following:

$$\left| \begin{array}{c} 0 > S_1(p-m) + S_2(p-m) - S_1 p \\ 0 > S_1(p-m) \end{array} \right.$$

It follows from the second inequality that we need ***p-m<0***.

The first inequality can be reworked further and one obtains

$$0 > S_1 p - S_1 m + S_2 p - S_2 m - S_1 p$$

$$m(S_1 + S_2) > S_2 p$$

And ultimately the solution to the system of inequalities is:

$$\left| \begin{array}{c} p < m.\dfrac{S_1 + S_2}{S_2} \\ p < m \end{array} \right.$$

Of the two conditions, the second is stronger (***p*** < ***m***). The first condition will always be true when the second is true, but not vice versa. This is because for each of the bars it will always be true that $\frac{S_1+S_2}{S_2} > 1$ - always the total number of customers of the two bars will be greater than the number of customers of either of the two bars, in this case the customers of the competing bar. Therefore, it will always be true that $m.\frac{S_1+S_2}{S_2} > m$ and if the condition $p < m$ is satisfied, the condition $m.\frac{S_1+S_2}{S_2} > p$ will also be satisfied.

For player B the condition for strict dominance will be similar:

$$\left| \begin{array}{c} S_2 p > S_1 q(p-m) + S_2 q(p-m) + S_2(1-q)p \\ S_2(1-q)p > S_2 q(p-m) + S_2(1-q)p \end{array} \right.$$

After similar revisions and simplifications, the following solution is obtained:

$$\left| \begin{array}{c} p < m.\dfrac{S_1 + S_2}{S_1} \\ p < m \end{array} \right.$$

Note: in the following text we will not detail all transformations leading to a simplification of expressions. A knowledge of elementary algebra is sufficient to enable any reader to do them himself.

Again the stronger of the two conditions is the second ***p*** < ***m***. It is also true again that the parameter *q* **has no effect** on the solution, which means that it does not matter exactly how large its value is and exactly how “loyal” the customers of the two bars are.

So, for the strategy of **not incurring** additional marketing costs to be the dominant strategy for either of the two players, a single condition must hold: ***p*** < ***m***. But if it is satisfied, both players will have one dominant strategy.

Does not this condition (***p*** < ***m***) contradict the sufficient condition for the existence of a single NE - that only one of the players has a strictly dominant strategy? Having two strictly dominant strategies is a "more than sufficient" condition.

No, because given a satisfied condition (***p*** < ***m***) it is impossible that only one of the two players has a dominant strategy. This is a case where the sufficient condition automatically extends to more than enough.

Is this logical? Yes, because if the extra cost of free peanuts is greater than the profit from one cup of beer ordered, this means a loss for each player with each cup of beer ordered. Since some part (q) of the bars' customers will still take advantage of a happy hour, this will reduce the profits of each of the two players, and therefore the better strategy for them is not to incur additional marketing costs. This NE will be unique, and the condition ***p*** < ***m*** **is more than sufficient** (compare with the prisoner's dilemma, where each player also has one strictly dominant strategy) for its existence and uniqueness.

Here we can return to one of the reasons why some clients do not take advantage of a happy hour. As we have said, it is possible that the extra serving of peanuts is only served when ordering 0.5l beer, but not for the smaller orders of 0.33l or 0.25l. Now it becomes clear why this is possible and rational - the profit from a 0.25l beer may be less than the extra cost of free peanuts, and then the bar owner will not offer peanuts with it either. Maybe he will give one serving of peanuts with the order of two 0.25l beers, but for those customers who only want to drink one this is not an attractive option and they will not be attracted. If peanuts are only offered on a 0.5L order, then only those drinking that much (or even more) beer will be potentially attracted, as long as some of the other reasons listed do not come into play.

As we will see later, this is not a necessary condition for the existence of NE at all (not unique equilibrium), and the distinction between a necessary and sufficient condition determines more interesting solution domains of the game.

In the ***p*** = ***m*** equality, there is also a dominance of the strategy of not incurring additional marketing costs over the strategy of incurring additional marketing costs for both players, but it dominates non-strictly (weakly). Because of the equality, the expressions in the payment matrix are greatly simplified:

iv. Payment matrices of the two players under condition p = m

Player A/ Player B	B1: Does not give free peanuts	B2: Gives free peanuts
A1: Does not give free peanuts	S_1p; S_2p	$S_1(1\text{-}q)p$; $S_2(1\text{-}q)p$
A2: Gives free peanuts	$S_1(1\text{-}q)p$; $S_2(1\text{-}q)p$	$S_1(1\text{-}q)p$; $S_2(1\text{-}q)p$

One can see that for the two players, the strategy of not incurring additional marketing costs dominates weakly the strategy of incurring additional marketing costs and again there is a ~~unique~~ NE, formed by the strategies of not incurring additional marketing costs. This follows from the fact that $0<q<1$ and it will always be true that $Sp > S(1\text{-}q)p$ for either of the two bars. For player A, the requirements that the strategy A1 "Does not give free peanuts" dominates the strategy A2 "Give free peanuts" will be weakly satisfied:

$$\begin{aligned} S_1p &> S_1(1-q)p \\ S_1(1-q)p &= S_1(1-q)p \end{aligned}$$

Similarly, for player B it will be true that strategy B1 ("Does not give free peanuts ") dominates strategy B2 (“Give free peanuts”) weakly/non-strictly:

$$\begin{array}{ccc} S_2 p & > & S_2(1-q)p \\ S_2(1-q)p & = & S_2(1-q)p \end{array}$$

But the pair of strategies {Give free peanuts; Give free peanuts} also satisfies the conditions to be considered a NE - either player can change his strategy to give free peanuts when he is convinced that it does not lead to the expected increase in customers, but only to a decrease in profit. Then the other player can with relief change his strategy from weakly dominated to weakly dominating and now the pair of strategies { Does not give free peanuts; Does not give free peanuts } will form the more profitable NE for both players.

This is a special case in which the payoff for both players decreases regardless of whether one or the other player gives free peanuts. As can be seen from the payment matrices of both players, only the customers who will order beer **outside** a happy hour bring them profits, and it does not matter if and how many customers the player who introduced a happy hour attracted.

The case is also interesting in that the presence of two non-strictly dominating strategies guarantees **indifference** for each player to the actions of the other player.

Furthermore, in the following it will always be true that the value of the parameter *q* does not affect the solution and it will no longer be explicitly emphasized that we truncate this number in the transformed expressions. However, we will always record the payoffs in their full form, with the *q* parameter included.

When examining the conditions for different solutions, we will always obtain some symmetry for player A and for player B, which means that results obtained for one player can be considered as proven for the other “by symmetry”. However, we will always prove them for both players, for completeness of exposition and so that no doubts arise that inconvenient expressions and results are omitted by the author.

The usual (but unproven) conclusion or more accurately - assertion in the “two bars” game is that it is a complete analogue of the “Prisoner’s Dilemma”, meaning that there is a single NE formed by the strategy pair {Give free peanuts; Give free peanuts}. We will therefore investigate the conditions for this to be the case. Again, we will first investigate the more than sufficient condition, i.e. that the strategy of making additional marketing expenditures is the dominant strategy for both players.

Condition the strategy to make additional marketing expenditures to be the dominant strategy for both players

This is the solution to the following system of inequalities:

$$\left| \begin{array}{c} S_1 p < S_1 q(p-m) + S_2 q(p-m) + S_1(1-q)p \\ S_1(1-q)p < S_1 q(p-m) + S_1(1-q)p \\ S_2 p < S_1 q(p-m) + S_2 q(p-m) + S_2(1-q)p \\ S_2(1-q)p < S_2 q(p-m) + S_2(1-q)p \end{array} \right.$$

The first two inequalities are for player A and the next two are for player B.

After analogous transformations the solution is obtained

$$\left| \begin{array}{l} m.\dfrac{S_1+S_2}{S_2} < p \\ 0 < S_1(p-m) \\ m.\dfrac{S_1+S_2}{S_1} < p \\ 0 < S_2(p-m) \end{array} \right.$$

Which is reduced to only three conditions, since the conditions for non-negativity of the two players’ profits less marketing costs ***(p*** > ***m***) coincide:

$$\left|\begin{array}{c} m.\dfrac{S_1+S_2}{S_2}<p \\ m<p \\ m.\dfrac{S_1+S_2}{S_1}<p \\ m<p \end{array}\right.$$

Unlike the previous case, here the stronger condition is not ***p*** > ***m***, because it will be satisfied when either of the other two conditions is satisfied.

Clearly, if holds $p > m.\frac{S_1+S_2}{S_1}$ and $p > m.\frac{S_1+S_2}{S_2}$, $p>m$ will always be true.

The strongest condition depends on which bar has **fewer** customers, so the condition for the strategy of making additional marketing expenditures to be the dominant strategy for both players, which is also **more than sufficient** condition for there to be unique NE in the strategy pair {Gives free peanuts; Gives free peanuts} looks like this:

$$p > m.\frac{S_1+S_2}{min\{S_1,S_2\}}$$

Since in general the number of customers in the two bars is different, the following conditions must be met

$$p > m.\frac{S_1+S_2}{min\{S_1,S_2\}} > m.\frac{S_1+S_2}{max\{S_1,S_2\}} > m$$

The more unequally the customers are distributed between the two bars, the more the profit from a beer must exceed the price of peanuts for the strategy of giving extra peanuts to be dominant for each of the two players and the pair of strategies {Give free peanuts; Give free peanuts} to form a single NE.

Only in the individual case in which the number of customers of the two bars is equal does the condition translate into the much simpler expression $p = 2m$. In the other cases $p > m.\frac{S_1+S_2}{min\{S_1,S_2\}} > 2m$.

Thus, we can formulate the condition for the existence of dominance of the strategy of making additional marketing expenditures for each of the players at all in a simpler and more understandable way: *'For the strategy of making additional marketing expenditures to be dominant for each of the two players, the profit from one beer must be* ***at least twice*** *the price of the peanuts that are offered for free*'.

For the consumer, this means that every time (in the competitive game for his money) he is offered something 'free' when buying something else, the profit from that 'something else' is **at least twice** the price of the 'free' gift.

If this condition is not satisfied, there will be no dominance of the strategy of making additional marketing expenditures for both players, but that does not mean that there will be no NE formed by the pair of strategies {Gives free peanuts; Gives free peanuts}, the two should not be confused. The profit from a beer must exceed the marketing costs by more than a factor of 2 (and not just exceed them) for this NE alone to exist, similar to the unique equilibrium in the Prisoner's dilemma.

Only one NE formed by the pair of strategies {Gives free peanuts; Gives free peanuts} can exist too in the intermediate case when the following condition is satisfied

$$m.\frac{S_1+S_2}{min\{S_1,S_2\}} > p > m.\frac{S_1+S_2}{max\{S_1,S_2\}} > m$$

This means that it is possible that the strategy of giving free peanuts is dominant only for one of the players, which is a necessary and sufficient condition for unique NE to exist. For which of the two players will this strategy be dominant? From the conditions it is obvious - this is the player whose bar has fewer customers. We will prove that this player has a dominant strategy of giving free peanuts and that there exists unique NE formed by the strategy pair {Gives free peanuts; Gives free peanuts}.

Let it be true that $S_2>S_1$ (player B has more customers than player A) and furthermore $m.\frac{S_1+S_2}{S_1} > p > m.\frac{S_1+S_2}{S_2} > m$. Then the dominant strategy for player A will be to give extra peanuts, i.e. he will be the active party trying to attract some of the opponent's customers. The following must be true:

$$\left| \begin{array}{c} S_1 p < S_1 q(p-m) + S_2 q(p-m) + S_1(1-q)p \\ S_1(1-q)p < S_1 q(p-m) + S_1(1-q)p \end{array} \right.$$

After all simplifications of the two inequalities we get

$$\left| \begin{array}{l} \frac{S_1+S_2}{S_1}.m < p \\ 0 < S_1 q(p-m) \end{array} \right.$$

and therefore really for player A the dominant strategy is to give extra peanuts. It remains to be seen which one is the best strategy for player B (his optimal response) if player A sticks to his dominant strategy. We need to compare his payoffs under the two possible strategies, but only for the case of player A's dominant strategy.

If player B uses the strategy {D**oes not give free peanuts**}, his payoff will be $S_2(1-q)p$ - he will lose some part (*q*) of his customers, but he will keep the profit level of one beer. As a result, his profit will decrease because of the reduced number of customers.

If player B uses the strategy {**Give free peanuts}**, his profit will be $S_2 q(p-m) + S_2(1-q)p$ - he will not lose any of his customers, but some of them will still take advantage of a happy hour and this will reduce the total profit from these customers. As a result, his profit will decrease at the expense of the decreased profit from the customers taking advantage of the happy hour. However, this option is preferable because even these customers will bring him profit:

$$S_2(1-q)p < S_2 q(p-m) + S_2(1-q)p$$

Therefore, the better strategy for him is also to introduce a happy hour and both players will do it: player A - because this is a dominant strategy for him, and player B - because it becomes a dominant strategy for him (it is also said that it is an optimal response to the strategy used by player A) when player A follows his dominant strategy.

The existence of a NE formed by the strategy pair {Gives free peanuts; Gives free peanuts} is possible without any dominant strategies existing at all. Here we were talking about the existence of a **unique** NE, because we will see later that under certain conditions two NEs will coexist.

So far we obtained more than sufficient conditions for the existence of unique alternative NEs, those formed by the pairs of strategies {Does not give free peanuts; Does not give free peanuts} and {Gives free peanuts; Gives free peanuts}. Open questions remain about the **necessary** conditions for the existence of these two NEs. These conditions are no longer derived from the dominance conditions, but from the conditions for a given pair of strategies to form a NE.

Condition the strategies to make additional marketing spending for both players to form a NE

This condition is the solution of the following system of inequalities:

$S_1(1-q)p < S_1 q(p-m) + S_1(1-q)p$
for player A and

$S_2(1-q)p < S_2 q(p-m) + S_2(1-q)p$
for player B

After the obvious simplification, the necessary condition for the existence of

NE formed by the strategy pair {Gives free peanuts; Gives free peanuts} is obtained:

$$S_1 q(p-m) > 0 \; and$$

$$S_2 q(p-m) > 0$$

Regardless of the values of the parameters *q, S_1* and S_2, only **p** > **m** must be true. This is also the **necessary** condition for the existence of the NE formed by the strategy pair {Gives free peanuts; Gives free peanuts}. If it is not satisfied, i.e., $\boldsymbol{p} \leq \boldsymbol{m}$, either the **more than sufficient** condition for existence of a unique NE formed by the strategy pair {Gives no free peanuts; Gives no free peanuts} will be satisfied, or two NEs will exist because then the strategies of not incurring additional marketing costs for both players will be non-strictly dominant strategies, as we have already demonstrated.

We can also say that a sufficient condition for making marketing expenditures is that they do not lead to losses or, in another formulation, *that the selling price of a beer is higher than its acquisition cost plus the marketing costs of a beer*.

We should emphasize that this is a necessary condition for the existence of the NE formed by the pair of strategies {Gives free peanuts; Gives free peanuts}, but this does not mean that the equilibrium will be unique. The same is true for the next condition.

Condition the strategies to not incur additional marketing costs for both players to form a NE

This condition is the solution of the following system of inequalities:

$S_1 p > S_1 q(p-m) + S_2 q(p-m) + S_1(1-q)p$ for player A and

$S_2 p > S_1 q(p-m) + S_2 q(p-m) + S_2(1-q)p$ for player B.

After transformation it is obtained

$$\left| \begin{array}{l} 0 > S_2 q(p-m) - S_1 qm \\ 0 > S_1 q(p-m) - S_2 qm \end{array} \right.$$

Again, the value of the parameter q has no effect on the solution, and the following **necessary** condition for the existence of a **non-unique** NE formed by the pair of strategies {No free peanuts; No free peanuts} is finally obtained:

$$\left| \begin{array}{l} p < m.\dfrac{S_1 + S_2}{S_2} \\ p < m.\dfrac{S_1 + S_2}{S_1} \end{array} \right.$$

One of the two conditions is stronger and this is the condition

$$p < m.\frac{S_1 + S_2}{max\{S_1, S_2\}}$$

In the general case, when the number of customers in each of the two bars is different, $p < m.\frac{S_1+S_2}{max\{S_1,S_2\}} < m.\frac{S_1+S_2}{min\{S_1,S_2\}}\}$ must be true.

In the particular case where the two bars have equal numbers of customers, this condition simplifies to the condition $p < 2m$.

As can be seen, the **necessary** condition does not exclude the possibility that the profit from one beer is less than the additional cost of a serving of peanuts (*m*), which was **more than sufficient** condition for the existence of the unique NE formed by the strategy pair{Does not give free peanuts; Does not give free peanuts}.

It is possible that the strategies of the two players to not incur additional marketing costs to form NE, **yet** these costs do not necessarily lead to losses.

More importantly, the **necessary** condition also does not exclude the possibility that the profit is greater than the additional cost, which was the **necessary** condition for the existence of the NE formed by the strategy pair {Gives free peanuts; Gives free peanuts}. It is possible, therefore, that $m < p < m.\frac{S_1+S_2}{max\{S_1,S_2\}}$ and then the conditions for the simultaneous existence of two NEs would be satisfied. We will prove this in the next section.

Condition for coexistence of two NEs

This condition is the solution to a more complex system of inequalities, since seemingly contradictory and incompatible conditions must be satisfied simultaneously.

We will consider each of the conditions forming the system of inequalities separately, with the necessary brief explanations. The existence of a solution of the system will imply the possibility of the simultaneous existence of two NEs and, on the contrary, if the system of inequalities is incompatible, this will be impossible.

A condition for the NE formed by the pair of strategies {Does not give free peanuts; Does not give free peanuts} to exist for player A is that the strategy of no additional marketing expenditure is preferable if player B follows his strategy of no additional marketing expenditure:

$$S_1p > S_1q(p-m) + S_2q(p-m) + S_1(1-q)p$$

The analogous condition for player B's strategy to not incur additional marketing costs to be preferable if player A sticks to his strategy of no additional marketing expenditures:

$$S_2p > S_1q(p-m) + S_2q(p-m) + S_2(1-q)p$$

A condition for the NE formed by the strategy pair {Gives free peanuts; Gives free peanuts} to exist for player A is that the strategy of making additional marketing expenditures is preferable if player B follows his strategy of making additional marketing expenditures:

$$S_1q(p-m) + S_1(1-q)p > S_1(1-q)p$$

The analogous condition for player B's strategy of making additional marketing expenditures to be preferable if player A sticks to his strategy of making additional marketing expenditures:

$$S_2q(p-m) + S_2(1-q)p > S_2(1-q)p$$

After simplifying the above expressions, the following system is obtained:

$$\left|\begin{array}{c} 0 > -S_1qm + S_2qp - S_2qm \\ 0 > S_1qp - S_1qm - S_2qm \\ S_1q(p-m) > 0 \\ S_2q(p-m) > 0 \end{array}\right.$$

After all simplifications and regroupings we get the following system of inequalities:

$$\begin{array}{c} m.\dfrac{S_1+S_2}{S_2} > p \\ m.\dfrac{S_1+S_2}{S_1} > p \\ p > m \\ p > m \end{array}$$

The final condition can be written like this:

$$m.\frac{S_1+S_2}{min\{S_1,S_2\}} > m.\frac{S_1+S_2}{max\{S_1,S_2\}} > p > m$$

As one can see, the possibility of two NEs coexisting is realized when $m.\frac{S_1+S_2}{max\{S_1,S_2\}} > p > m$, which is possible since this interval is not and cannot be empty. This possibility exists because of the mismatch of **necessary** and **sufficient** conditions for the existence of a NE.

Since in the interval $m, m.\frac{S_1+S_2}{max\{S_1,S_2\}}$ both conditions are satisfied simultaneously, the existence of two NEs is possible at any value of *p* falling within it.

We can summarize the necessary and sufficient conditions for the existence of NE in the following table:

v. Necessary and sufficient conditions for the existence of NE

NE/condition	sufficient	necessary
Pair strategies {Does not give free peanuts; Does not give free peanuts}	$p \leq m$	$p < m.\dfrac{S_1 + S_2}{max\{S_1, S_2\}}$
Pair strategies {Gives free peanuts; Gives free peanuts}	$p > m.\dfrac{S_1 + S_2}{min\{S_1, S_2\}}$	$p \geq m$

As can be seen, the difference between profit per beer and marketing costs (the price of free peanuts) is crucial to which NE the two players will be in.

If the marketing costs are greater than the profit from one beer, each glass sold will bring losses to the bar, and the more customers that bar attracts, the greater those losses will be. In a hypothetically pure model in which the two bars offer only beer and peanuts, the introduction of a happy hour will lead to losses for one bar and reduced profits for the other bar. The bar that realizes losses with each glass of beer sold during a happy hour then reduces those losses as the duration of a happy hour decreases and will minimize them when it reduces that duration to 0. The dominant strategy for both players will be to not offer free peanuts, and this pair of strategies will form the unique NE.

For the strategy of giving free peanuts to become usable at all, their cost must not exceed the profit from the glass of beer with which they are served. Therefore, again, the difference between this price and the profit from a glass of beer determines whether free peanuts will be served with every glass of beer (0.25l, 0.33l, 0.5l, 1l) or only with certain orders (e.g. - 2 glasses of 0.25l or 1 glass of 0.5l but not one glass of 0.33l).

Conversely, if one ordered glass of beer brings a very high profit, exceeding **at least twice** the price of free peanuts, the dominant strategy for both players will be to try to attract additional customers through a happy hour and the pair of strategies to offer free peanuts will become dominant for both, where it forms the unique NE.

The economic logic here is: since a glass of beer brings in so much profit, why not give up some of that profit since each additional beer sold (from the competitor's extra customers attracted) will bring in more profit? And, even if the competitor also introduces a happy hour, it **should be** countered in the same way so that some of the beers sold and the profit from them is not lost, even at the cost of reducing profit, because that reduction will be the lesser evil.

When only the necessary conditions for the existence of a NE are simultaneously satisfied, but none of the sufficient conditions are satisfied, there will be two NEs, the pair of strategies to offer free peanuts and the pair of strategies not to offer free peanuts.

It can also be seen from the table that it is possible to have a situation in which the **necessary** condition for the existence of the unique NE formed only by the strategy pair {Does not give free peanuts; Does not give free peanuts} is not satisfied, but the **sufficient** condition for the existence of the unique NE formed only by the strategy pair {Gives free peanuts; Gives free peanuts} is also not satisfied. This would imply that the following conditions are simultaneously satisfied:

$$m < m.\frac{S_1 + S_2}{max\{S_1, S_2\}} < p < m.\frac{S_1 + S_2}{min\{S_1, S_2\}}$$

But this does not mean that there is some interval of values for the profit - a “no man’s land” in which none of the NEs studied so far will exist. It is enough to see from the table that the **necessary** condition for the existence of the NE formed by the strategy pair {Gives free peanuts; Gives free peanuts} will be satisfied and therefore this NE will exist and it will be unique.

It remains an unexplored question whether it is possible for NEs to exist in mixed strategies and NEs in pairs of “asymmetric” strategies {Does not give free peanuts, Gives free peanuts} and/or {Gives free peanuts, Does not give free peanuts}, which would imply that the two players use different pure strategies in the competitive struggle. We will first investigate the existence conditions of these two NEs in pure strategies.

Existence condition for NE formed by the pair of strategies {Does not give free peanuts, Gives free peanuts}

This condition is the solution of the following system of inequalities:

$S_1(1-q)p > S_1q(p-m) + S_1(1-q)p$ for player A and

$S_1q(p-m) + S_2q(p-m) + S_2(1-q)p > S_2p$ for player B.

After the obvious simplifications and transformation we get

$$\left| \begin{array}{c} 0 > S_1q(p-m) \\ S_1q(p-m) + S_2q(p-m) - S_2qp > 0 \end{array} \right.$$

Again, the value of the parameter q can be neglected and the following system of condition-inequalities is finally obtained:

$$\left| \begin{array}{c} m > p \\ S_1(p-m) > S_2m \end{array} \right.$$

This system of conditions is incompatible - if $m > p$ is true from the first inequality, then in the second inequality the left hand side $S_1(p-m)$ will be negative, but then it cannot be greater than the non-negative number on the right hand side of the inequality. The losses of the first bar (a negative number) cannot be greater than the marketing costs in the competing bar (a non-negative number). Hence, there will be no NE formed by the strategy pair {Does not give free peanuts; Gives free peanuts}. We can say that an analogous “by symmetry” result should hold for the equilibrium formed by the strategy pair {Gives free peanuts; Does not give free peanuts}, which we will prove briefly in the next section.

Condition for the existence of NE formed by the pair of strategies {Gives free peanuts; Does not give free peanuts}

This condition is the solution of the following system of inequalities:

$S_1q(p-m) + S_2q(p-m) + S_1(1-q)p > S_1p$ for player A and

$S_2(1-q)p > S_2q(p-m) + S_2(1-q)p$ for player B.

After the obvious simplifications and transformation we get

$$\left| \begin{array}{c} S_1q(p-m) + S_2q(p-m) > S_1qp \\ 0 > S_2q(p-m) \end{array} \right.$$

Again, the value of the parameter q can be neglected and the following system of condition-inequalities is finally obtained:

$$\left| \begin{array}{c} S_2(p-m) > S_1m \\ m > p \end{array} \right.$$

Similarly, these two inequalities also form incompatible domains of admissible solutions as in the previous case, and hence there can be no NE formed by the strategy pair {Gives free peanuts; Does not give free peanuts}.

Hence, under no conditions is it possible for there to exist a NE formed by divergent (different or asymmetric) strategies for the two players.

Existence of an optimal mixed strategy

The existence or not of an optimal mixed strategy in this game is fundamental.

First of all, the game between the two players can be played many times, which is a requirement to talk about mixed strategy at all. They have the freedom to introduce a happy hour or not, and to decide at which exact hour of the bar's operation to do so.

In the second place, the optimal mixed strategy can answer the question "How many hours should a happy hour last?". So far, we have considered a happy hour to be a pure strategy, but our view of it can be extended to consider a happy hour as a mixed strategy in which free peanuts are served only during one particular hour of the bar's operating hours and are not served during the rest of the time. Assuming the bar is open N hours a day, this mixed strategy would look like $\{\frac{N-1}{N}, \frac{1}{N}\}$. Then the existence of an optimal mixed strategy, conditional on the parameters of the game, will yield the optimal happy hour duration for each of the players.

On the contrary, if it is proved that no optimal strategy exists for any player, this would imply that the game has solutions only in pure strategies. We consider the conditions for the existence of an optimal mixed strategy for each of the two players in turn.

Player A's optimal mixed strategy

Here we investigate the existence or not of an **optimal mixed strategy** for player A. Such a strategy may exist when the conditions for dominance of either strategy are not satisfied. If one of the strategies is dominant for player A, no mixed strategy will exist for him and unique NE will exist in pure strategies.

For the strategy "Does not give free peanuts" **to be not** dominant for player A the condition ***p* ≤ *m* must be not** satisfied, i.e. ***p* > *m*** is true.

For the strategy "Gives free peanuts" only for player A **to be not** dominant, the dominance condition set by the inequality system **must not be** satisfied:

$$\left|\begin{array}{c} S_1q(p-m) + S_2q(p-m) + S_1(1-q)p > S_1p \\ S_1q(p-m) > 0 \end{array}\right.$$

These expressions are already familiar to us, so we will give the final results of their transformation:

$$\begin{array}{c} S_1(p-m) + S_2(p-m) > S_1p \\ S_1q(p-m) > 0 \end{array}$$

Since ***p*>*m*** must be true, the second inequality will be satisfied and it remains that the first one is not true, which means is not true $m.\frac{S_1+S_2}{S_2} < p$.

Hence, it must be true that $m.\frac{S_1+S_2}{S_2} > p > m$, which means that **it is not in principle impossible** for NE to exist in mixed strategies.

Therefore, we will write down and search for the optimal mixed strategy in general. We look for probabilities *x* and *1-x* such that the expected payoff of player A will not depend on the strategy chosen by player B, also mixed in the general case.

The solution of the equation must be found (see for example (Knowles, 1989) pp. 567-570)

$$xS_1p + (1-x)[S_1q(p-m) + S_2q(p-m) + S_1(1-q)p] =$$
$$= xS_1(1-q)p + (1-x)[S_1q(p-m) + S_1(1-q)p]$$

After regrouping and simplification, we obtain

$$(1-x)[S_2(p-m)] = -xS_1p$$

This equation has the following solution for *x*:

$$x = \frac{S_2(p-m)}{S_2(p-m) - S_1p}$$

The numerator will always be a positive number when the condition ***p***>***m*** is satisfied.

There are two possibilities for the sign of the denominator:

$S_2(p-m) - S_1p > 0$ and then $S_2(p-m) > S_2(p-m) - S_1p > 0$ will always be true and therefore $x = \frac{S_2(p-m)}{S_2(p-m)-S_1p} > 1$

$S_2(p-m) - S_1p < 0$ and therefore $x = \frac{S_2(p-m)}{S_2(p-m)-S_1p} < 0$

In either case, the value of *x* is outside the definitional domain for probability and there is no optimal mixed strategy for player A.

The possibility that there exists an optimal mixed strategy for player B is investigated in the same way.

Player B's optimal mixed strategy

Player B's mixed strategy is the solution to the equation

$$xS_2p + (1-x)[S_1q(p-m) + S_2q(p-m) + S_2p(1-q)] = \\ = xS_2p(1-q) + (1-x)[S_2q(p-m) + S_2p(1-q)]$$

Similar to the previous case, the solution for x is

$$x = \frac{S_1(p-m)}{S_1(p-m) - S_2p}$$

Again, this solution is similarly found to be outside the definitional domain of probability.

The following final table summarizes the results depending on the magnitude of the gain:

iv. Summary

p has value	Condition is satisfied	Result
p<m	Strict dominance of the "Does not give free peanuts" strategy, more than sufficient condition for unique NE	There is only one NE formed by the strategy pair {Does not give free peanuts; Does not give free peanuts}
p=m	Weak (non-strict) dominance of the "Does not give free peanuts" strategy, a sufficient condition for NE	There is one NE formed by the strategy pair {Does not give free peanuts; Does not give free peanuts} and a second formed by the strategy pair {Gives free peanuts; Gives free peanuts}
p>m	A necessary condition for the NE formed by the strategy pair {Gives free peanuts; Gives free peanuts}	There is at least one NE formed by the strategy pair {Gives free peanuts; Gives free peanuts}
$m.\frac{S_1+S_2}{max\{S_1,S_2\}} > p > m$	A necessary condition for the NE formed by the strategy pair {Does not give free peanuts; Does not give free peanuts}	There is one NE formed by the strategy pair {Does not give free peanuts; Does not give free peanuts} and a second formed by the strategy pair {Gives free peanuts; Gives free peanuts}

p has value	Condition is satisfied	Result
$m.\frac{S_1 + S_2}{min\{S_1, S_2\}} > p > m.\frac{S_1 + S_2}{max\{S_1, S_2\}} > m$	A necessary condition for the NE formed by the strategy pair {Gives free peanuts; Gives free peanuts}	There is only one NE formed by the strategy pair {Gives free peanuts; Gives free peanuts}
$p > m.\frac{S_1 + S_2}{min\{S_1, S_2\}}$	Strict domination of the “Gives free peanuts” strategy, more than a sufficient condition for unique NE	There is only one NE formed by the strategy pair {Gives free peanuts; Gives free peanuts}

We can see from the table that for every possible difference between profit and marketing costs, there exists at least one NE in pure strategies, and no such difference can be found for which no such NE exists. This also explains why, under any conditions, no NE would exist in mixed strategies.

Conclusion

Although formally both the Prisoner’s Dilemma and the Two-Bar Dilemma are games of the form D (2,2), they are fundamentally different.

The first game is organized by a **rational force** in accordance with the goals of the organizer, the judiciary. Its payoffs are chosen so that one dominant strategy is always guaranteed for each of the players, and it is exactly the strategy the judiciary needs.

The second game is “organized” by an **unreasonable force** - competition - which has no objectives. The players in the game have clear goals - greater profit - but the “organizer” does not. The payments in it are what they are in the specific situation, depending on the differences between profit and marketing costs and customer distribution between the two players. Therefore, the players may not have a dominant strategy or the dominant strategy may change, also depending on the differences between profit and marketing costs and customer distribution between the two players. From these differences follow different possible solutions of the players, not a unique and invariably over time one.

“Customer loyalty”, as measured by the proportion of customers who would not change their bar for an extra free serving of peanuts, is in fact irrelevant to the conclusions drawn and research in this direction and in this context can therefore be considered to be self-serving and unpromising. Much more interesting, in the author’s opinion, are studies explaining the personal motives for customers to be “loyal”.

If the marketing costs and profits of each player are considered as dynamic quantities changing over time, then the strategies of the two players and the equilibria in which the game will be at any point of time should also be seen as changing over time.

It is a dangerous fallacy to believe that the two-bar game is a complete analogue of the prisoner’s dilemma, but only transposed to economics. From this fallacy can follow patterned and incorrect decisions and two types of incorrect conclusions about the role of marketing costs in the competition.

The first type is the conclusion that marketing expenditure is some kind of magic bullet, applicable in any competitive situation and always leading to positive results. The fallacy of this type is summarized as "marketing at any cost".

The second type is the conclusion that marketing costs are useless in the competition and can be ignored. The wrong action in this type of error is summarized as "no marketing costs".

Both types of conclusions are equally wrong if one does not proceed to a careful analysis of the situation. Game Theory gives us the means for such analysis and for appropriate conclusions and actions.

Works Cited

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Author:

Assoc. Prof. Dr. Georgi Kiranchev - University of National and World Economy - Sofia, Bulgaria